Modal Diversity for Robust Free-Space Optical Communications

Mitchell A. Cox,[1,*] Ling Cheng,[1] Carmelo Rosales-Guzmán,[2] and Andrew Forbes[2]

[1] *School of Electrical and Information Engineering, University of the Witwatersrand, Johannesburg, South Africa*

[2] *School of Physics, University of the Witwatersrand, Johannesburg, South Africa*

(Received 2 February 2018; revised manuscript received 11 April 2018; published 15 August 2018)

Free-space communication links are severely affected by atmospheric turbulence, which causes degradation in the transmitted signal. One of the most common solutions to overcome this is to exploit diversity. In this approach, information is sent in parallel over different paths using two or more transmitters that are spatially separated, with each beam therefore experiencing different atmospheric turbulence, lowering the probability of a receive error. In this work we propose and experimentally demonstrate a alternative method of diversity based on spatial modes of light, which we call modal diversity. We remove the need for a physical separation of the transmitters by exploiting the fact that spatial modes of light experience different perturbations, even when traveling along the same path. For this proof-of-principle, we select identical radius modes from the Hermite-Gaussian and Laguerre-Gaussian basis sets and demonstrate an improvement in bit error rate by up to 54% without increasing the total transmit power or receive aperture radius.

DOI: 10.1103/PhysRevApplied.10.024020

I. INTRODUCTION

Free-space optical (FSO) communication has been the subject of significant research in recent years due to the impending capacity crunch [1]. The use of FSO communications as an alternative to radio is attractive because of ultrahigh-bandwidth capabilities when technologies such as mode-division multiplexing (MDM) are employed [2–5]. In MDM, several orthogonal modes are used, each carrying a separate data stream to multiply the overall data rate of a system. The use of the azimuthal subset of the Laguerre-Gauss ($LG_\ell^{p=0}$) basis, specified by mode index ℓ, has been well studied due to the simplicity of creation and (de)multiplexing techniques. The use of the radial LG index, p, is uncommon but is required to harness the full capacity of the basis [6,7]. The Hermite-Gauss (HG_n^m) basis can also be used for MDM and is specified by mode indices n and m, and beyond scalar modes, cylindrical vector-vortex (CVV) modes have also been the subject of attention for optical communications in both free-space and optical fibers [8–10].

Upon propagation through the atmosphere, turbulence distorts the wavefront, amplitude, and phase of the launched modes. Amplitude and phase fluctuations lead to so-called fading errors in a communication system with a certain probability and hence deteriorate the overall performance. In an MDM system these distortions also cause correlation between channels, leading to crosstalk that usually results in further communication errors [11,12]. These effects are dependent on the path that the beam propagates through and so when several statistically independent paths are used, due to a separation of at least r_0 (the Fried parameter, which is a measure of the transverse distance over which the refractive index is correlated [13]), the probability of error is reduced [14]. This is the basis for what is known as diversity and the so-called diversity gain can often be predicted by using probabilistic models applied to a range of mode types [12,15–20]. Diversity is considered the complement to multiplexing, reducing errors rather than directly increasing capacity [21]. While optical MDM has been extensively studied, diversity, and in particular modal diversity, has received little attention. This is not the case in fields such as radio communications, where diversity is harnessed as a matter of course [22,23].

In this work, we hypothesize and then experimentally demonstrate that a carefully chosen combination of identical radius HG and LG modes, each transmitted colinearly at a lower intensity to conserve the total transmit power, will result in a diversity gain; thus this alleviates the issue of large (or multiple) transmit and receive apertures while also significantly improving the robustness of the channel.

In the following section, a generalized model for the diversity channel is provided. An explanation of the difference between conventional multiplexing and diversity is also given. Section III describes the specific choice of modes required to achieve diversity without r_0 aperture separation, as well as the experimental setup. Results of the experiment and a discussion thereof can be found in Sec. IV and the conclusion is in Sec. V.

*mitchell.cox@wits.ac.za

II. CHANNEL MODEL

In a typical, uncoded diversity model, multiple (N) transmitters (lasers) transmit identical signals, $x_i(t)$, at the same time with amplitude g_i. In comparison to a single-transmitter case, the amplitude of each transmitter is scaled by $1/N$ to conserve overall transmit power. These signals propagate through separate channels represented by channel impulse responses, $h_i(t)$. This impulse response is typically modeled using a so-called fading probability distribution. The signal is then detected by a single receiver (photodiode) with receiver sensitivity r. The resulting received signal, $y(t)$, is then found from

$$y(t) = \frac{r}{N} \sum_{i}^{N} g_i x_i(t) * h_i(t) + n(t), \qquad (1)$$

where an additive white Gaussian noise (AWGN) component, $n(t)$, may be incorporated and asterisk is a convolution operator.

Traditionally, the term separate channels means physically distinct paths, which in atmospheric turbulence requires a physical separation of at least r_0 (atmospheric coherence length). This is because atmospheric turbulence results in spatially distributed random changes in the refractive index of air, which cause random distortions to the spatial profile of a beam wavefront [13]. In modal diversity we interpret separate channels as distinct modes with differing behavior in turbulence *without* being separated by a distance of r_0. We illustrate this concept in Fig. 1.

If the channel impulse responses are statistically independent, as well as the noise, then we can simply summarize the overall probability of an error in the diversity case as

$$\Pr[E_{\text{diversity}}] = \prod_{i} \Pr[E_i], \qquad (2)$$

where the probability of an error occurring for the ith channel is defined as $\Pr[E_i]$. This always results in a lower probability of error than a single-channel case (since $\Pr[E_i] \leq 1$). This particular diversity scheme is similar to equal gain combining (EGC), which is well known in radio communications [24] and is in fact one of the reasons why we find multiple antennas on WiFi routers, for instance. The pertinent point is that if the different channels, whether formed by spatial modes or spatial paths, are not statistically independent then we can expect that there will be no gain in terms of robustness, measurable by bit rate error (BER) or outage probability.

III. EXPERIMENT

A. Choice of modes for diversity

In Ref. [25], diversity was simulated using LG modes with a large mode spacing, with indices $\ell = +1, +8$, and $+15$. The results showed an improvement to the outage probability, or robustness, of a FSO link. Using modes from the same set necessitates differing (larger) mode orders, with the result that the receive aperture becomes impractically large with long propagation distances.

We address this issue by using identical order yet orthogonal HG_n^m and LG_ℓ^p beams, motivated by the fact that HG modes are robust to tip and tilt, which are the primary aberrations of atmospheric turbulence [26]. Beams with the same mode order have the same propagation parameters, resulting in the same field size at the receiving aperture (or detector). The order of the beams is given by $N = n + m = 2p + |\ell|$ for HG and LG beams respectively. An example of this are the modes in Fig. 1.

The completeness property of both bases allow us to express any element of one basis as a linear combination of elements from the other basis using the transformation

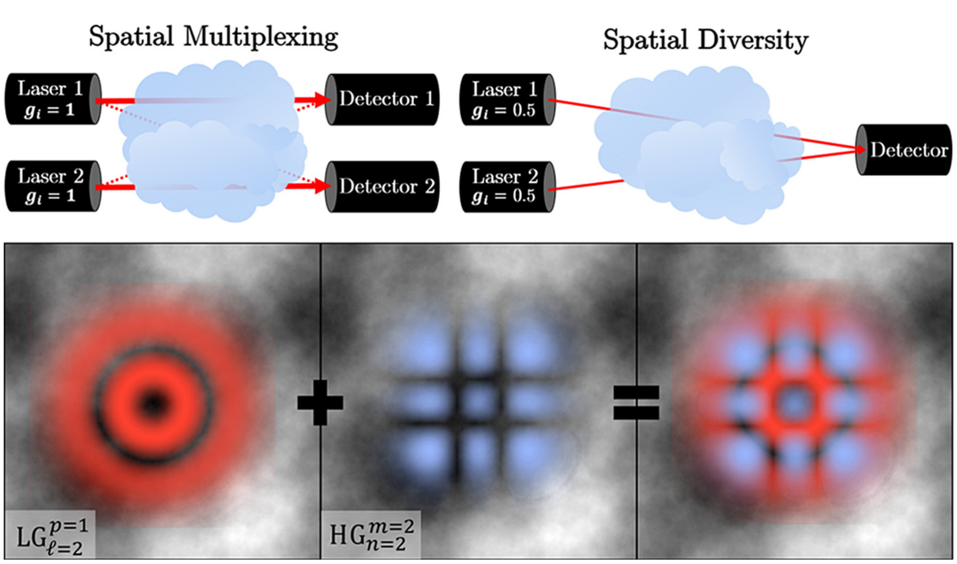

FIG. 1. Illustrative comparison of spatial multiplexing that is used for increased bandwidth (top left), conventional spatial diversity that is used for increased robustness (top right), and our modal diversity scheme that does not require path separation (bottom). In conventional diversity the transmitted beams, irrespective of mode, take different paths separated by some distance greater than r_0. In modal diversity, different modes are used with half the original transmit power (g_i) but with each traveling the same path: the separation is in mode space not physical space.

relations below [27]:

$$\mathrm{LG}_{n,m}(x,y,z) = \sum_{k=0}^{N} i^k b(n,m,k)\,\mathrm{HG}_{N-k,k}(x,y,z), \quad (3)$$

$$b(n,m,k) = \left[\frac{(N-k)!k!}{2^N n!m!}\right]^{1/2} \frac{1}{k}\frac{d^k}{dt^k}[(1-t)^n(1+t)^m]|_{t=0}. \quad (4)$$

The LG modes are written in terms of n and m, which are indices typically used for HG modes. Traditionally, LG modes are given in terms of azimuthal index ℓ and radial index p, which can be recovered as $\ell = n - m$ and $p = \min(n,m)$. For example, the LG_2^1 mode may be written as

$$\mathrm{LG}_2^1 = \frac{1}{2}\mathrm{HG}_4^0 - \frac{i}{2}\mathrm{HG}_3^1 + 0\times\mathrm{HG}_2^2 - \frac{i}{2}\mathrm{HG}_1^3 - \frac{1}{2}\mathrm{HG}_0^4. \quad (5)$$

Notice that the HG_2^2 component has a zero weighting, making the HG_2^2 mode orthogonal to the LG_2^1 mode. Figure 2 shows an experimental verification of this orthogonality. Conveniently, both of these modes have an order of $N = 4$; however, not all LG modes contain an orthogonal HG mode with the same mode order and vice versa. Similarly, HG_4^4 is orthogonal to LG_6^1 with $N = 8$. Both of these mode sets are tested in the experiment.

B. Experimental setup

The experimental setup consists of two laser diodes that are individually modulated with identical on-off keying signals. In this modulation scheme, which is common in FSO systems, the presence of light (on) represents a binary 1 and the absence of light (off) is a binary 0. For demodulation, a threshold is used to determine whether the received signal is off or on. As mentioned in the introduction, atmospheric turbulence will randomly modify the intensity of the signal by attenuation or crosstalk, which may lead to incorrect detections and subsequent errors. The modulation is performed at low bandwidth using a custom Arduino-based system at approximately 1 kbit/s. A high-bandwidth demonstration using expensive telecommunications equipment is deemed unnecessary for the purposes of this investigation as it is a proof of concept only.

As shown in Fig. 3, each beam is transformed using a spatial light modulator (SLM) into either an LG or HG mode. The first diffraction order from each SLM is spatially filtered and both beams are then combined using a beam splitter to propagate colinearly. Kolmogorov turbulence and modal decomposition are performed by another SLM [28,29]. Finally, the resulting beam is filtered using a 50-μm precision pinhole after which the intensity is measured by a photodiode for demodulation. The experiment is performed with varying turbulence strengths corresponding to a range of atmospheric coherence lengths (r_0) from 0.1 to 17 mm. These coherence lengths correspond to the Strehl ratios 0.01 to 0.9 when applied to the beams with mode order $N = 4$. For each turbulence strength and each mode, one million random bits are tested against 1024 random Kolmogorov phase screens. The BER is then determined for each test case. Each mode is first tested individually at a specific intensity in a conventional single-channel configuration to determine baseline BER performance for the experimental setup without diversity. Transmit diversity is then tested by transmitting a random bit stream over the HG and LG modes simultaneously. It is important to note that in the diversity case half of the single-channel intensity is used for each transmitter, resulting in the same total intensity as in the single-channel case at the receiver. This is done to ensure a fair SNR comparison.

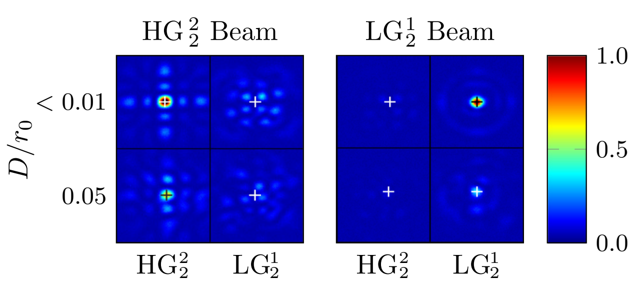

FIG. 2. Experimental demonstration of the orthogonality of HG_2^2 and LG_2^1 modes with no turbulence (SR = 1.0 or $D/R_0 < 0.01$) and slight turbulence (SR = 0.95 or $D/r_0 = 0.05$) with measurement points shown as +. On the left, only an HG_2^2 is transmitted and it is clear that there is no LG_2^1 component after modal decomposition. Similarly, on the right there is an LG_2^1 component but no HG_2^2 component in the decomposed beam.

IV. RESULTS AND DISCUSSION

BER results versus turbulence strength (r_0) are shown in Figs. 4 and 5. The BER of the diversity case is on average 23% better than that of the LG single-channel case for almost all turbulence strengths. At the weakest turbulence strength, the improvement in diversity BER over both single-channel cases is 54%. Due to the fact that the laser intensities are normalized, this can only be due to a diversity gain. It should be noted that the measured BER values are worse than what would normally be expected due to SLM flickering at a frequency close to that of the signal, which introduces noise and thus degrades the SNR. This issue does not affect our diversity claims, however; in a commercial communication system, BERs with an order of magnitude improvement over the results presented here can be expected.

Interestingly, the HG case is noticeably better than the LG case at medium turbulence strengths. Given that tip

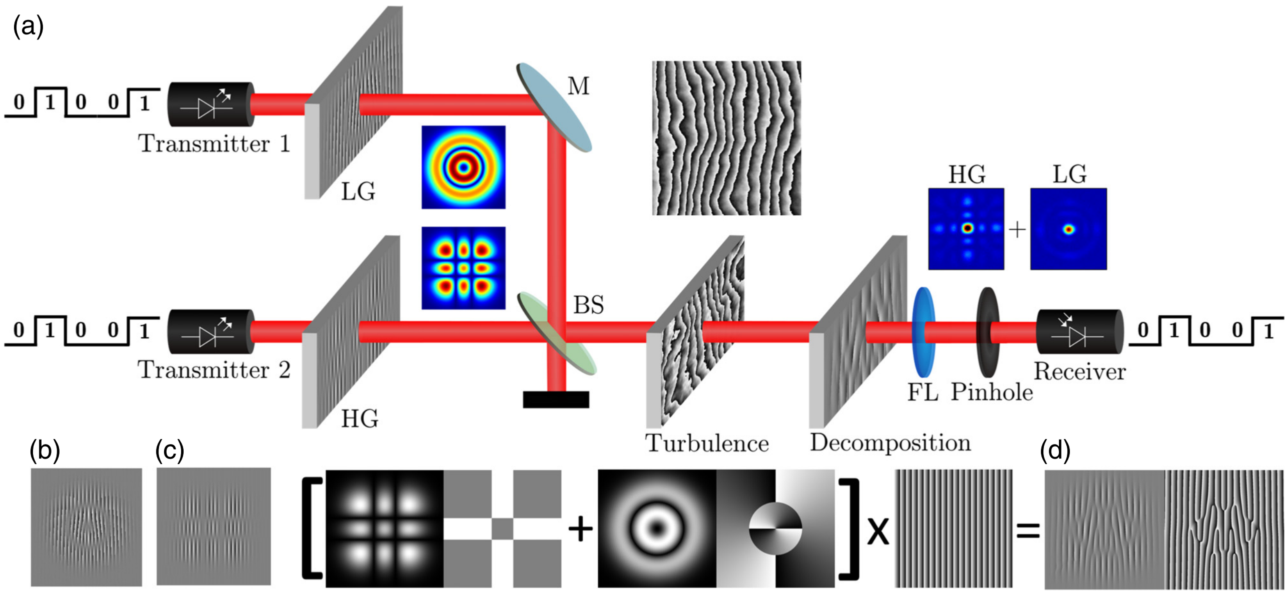

FIG. 3. Experimental setup showing two transmitters and a single receiver for transmit diversity with example turbulence as an inset (a). The holograms along the bottom are for (b) generating an $\mathrm{LG}_{\ell=2}^{p=1}$ beam, (c) generating an $\mathrm{HG}_{n=2}^{m=2}$ beam, and (d) superposition for decomposition of the generated LG and HG beams to a single detector. The flat gray areas on the holograms is due to complex amplitude modulation.

and tilt is the dominant aberration in atmospheric turbulence, this result agrees with the findings in Ref. [26], which demonstrate that HG modes are more resilient to tip and tilt aberrations than LG modes. At weaker turbulence strengths, characterized by smaller D/r_0 values, the difference between the HG_2^2 and LG_2^1 is no longer clearly visible where the difference between the HG_4^4 and LG_6^1 is quite prominent. The exact reason for this difference is the subject of further study, but we assume that this difference is due to the fact that in comparison to lower-order modes, the wavefronts of the higher-order modes have a larger diameter as well as more phase variation and therefore will interact with turbulence in a more dissimilar manner. It is for this reason that modal diversity is possible. The diversity gain for the lower-order modes is indeed still present because of the independent nature of the effect of turbulence on the HG and LG modes.

It is interesting to note that in the $N = 4$ case the BER converges to the worst case of 0.5 at $D/r_0 > 1$, as expected; however, in the $N = 8$ case this only happens

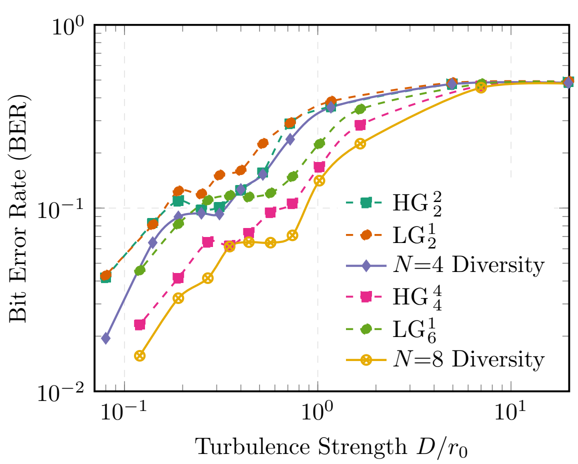

FIG. 4. Bit error rate of modes with order $N = 4$ and 8, where $D = 1.4$ and 1.98 mm, respectively, with varying turbulence strength showing a clear improvement for the diversity cases. Larger modes are expected to experience lower turbulence in comparison to smaller modes.

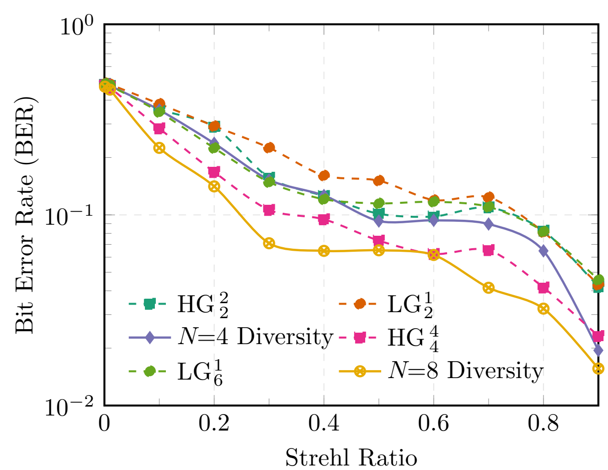

FIG. 5. Bit error rate of modes with order $N = 4$ and 8, with varying turbulence strength in terms of the Strehl ratio for $D = 1.4$ mm. The diversity case is typically better than the nondiversity cases with significant gains in weaker turbulence.

TABLE I. Example relations between D/r_0 and the equivalent propagation distance, z, with $C_n^2 = 10^{-14}$ $\mathrm{m}^{-2/3}$ for selected BERs according to Eq. (7) for the LG and diversity cases.

Bit error rate	0.04	0.08	0.15
LG_2^1 D/r_0 (mm)	0.08	0.14	0.31
Diversity D/r_0 (mm)	0.10	0.19	0.52
LG_2^1 distance (km)	0.24	0.55	2.13
Diversity distance (km)	0.37	0.93	5.06
Distance gain (%)	54	71	137

at approximately $D/r_0 > 6$. This indicates that even in stronger turbulence, where a conventional single-mode system is overcome by errors, there is still a margin provided by the diversity, which may prove useful when engineering a FSO link.

These results can be put into context by calculating the effective propagation distance gain at a specific BER [28]. If Kolmogorov turbulence is assumed, then we can write r_0 as a function of C_n^2, the refractive index structure parameter, the wavelength, λ, and the propagation distance, z:

$$r_0 = 0.185 \left(\frac{\lambda^2}{C_n^2 z} \right)^{3/5}. \tag{6}$$

This equation can then be solved for z, resulting in

$$z \approx \frac{0.0600647\lambda^2}{C_n^2 r_0^{5/3}}. \tag{7}$$

In this experiment, $\lambda = 660$ nm is used for ease of alignment as opposed to a standard telecommunications wavelength of 850 nm or even 1550 nm, for example. A typical value for C_n^2 is 10^{-14} $\mathrm{m}^{-2/3}$ in strong turbulence [28]. Three arbitrary BERs are selected and in Table I the corresponding D/r_0 values are provided. From Eq. (7), the corresponding theoretical propagation distances are calculated and shown in the table. It is clear that significant improvements in propagation distance are possible using modal diversity. In a well-engineered system, with the inclusion of additional techniques such as forward-error correction, robust modulation and interleaving, as well as sensitive low-noise detectors and laser drivers, the BER performance will be improved significantly.

V. CONCLUSION

Traditionally, diversity has been theoretically and experimentally demonstrated with a r_0 (or greater) separation between transmit and/or receive apertures. In this work we extend the concept of diversity to the modal diversity case, where a physical separation r_0 is not required, and achieve significant diversity gain. Based on the proof-of-principle results presented in this letter, our technique is able to increase the allowable propagation distance (determined by a certain error rate) by a significant margin at both strong and weak turbulence strengths. This finding can be used to significantly improve future FSO communication links in terms of range and reliability.

Our approach makes use of two or more orthogonal copropagating beams from different modal bases to take advantage of the fact that modes with different spatial profiles will interact differently with turbulent space. Although modal diversity can also be achieved using sufficiently spaced LG modes [25], the required receive aperture size may become prohibitive. In our approach, it is possible to choose orthogonal modes with the same order and thus the same size, negating the need for larger receive apertures than would otherwise be required.

A further implication of the use of different mode types is that the copropagating modes may experience a lower degree of correlation than neighboring adjacent modes within a single basis. This hypothesis should be tested in the future, but this may indeed have interesting consequences and provide significant benefits to MDM or when multiple-input and multiple-output signal processing is used. We speculate that it should be possible to formalize the concept of separate in terms of a newly defined modal distance, r_M, akin to the previous definition in terms of a physical distance, r_0. That is, how far would two modes have to be separated by in mode space, r_M, in order to ensure the same diversity gain as a physical separation of r_0?

ACKNOWLEDGMENT

The financial assistance of the National Research Foundation (NRF), the Claude Leon Foundation and CONACyT towards this research is acknowledged.

[1] David J. Richardson, Filling the light pipe, Science **330**, 327 (2010).

[2] Graham Gibson, Johannes Courtial, Miles J. Padgett, Mikhail Vasnetsov, Valeriy Pas'ko, Stephen M. Barnett, and Sonja Franke-Arnold, Free-space information transfer using light beams carrying orbital angular momentum, Opt. Express **12**, 5448 (2004).

[3] Jian Wang, Jeng-Yuan Yang, Irfan M. Fazal, Nisar Ahmed, Yan Yan, Hao Huang, Yongxiong Ren, Yang Yue, Samuel Dolinar, Moshe Tur, and Alan E. Willner, Terabit free-space data transmission employing orbital angular momentum multiplexing, Nat. Photonics **6**, 488 (2012).

[4] A. E. Willner, H. Huang, Y. Yan, Y. Ren, N. Ahmed, G. Xie, C. Bao, L. Li, Y. Cao, Z. Zhao, J. Wang, M. P. J. Lavery, M. Tur, S. Ramachandran, A. F. Molisch, N. Ashrafi, and S. Ashrafi, Optical communications using orbital angular momentum beams, Adv. Opt. Photonics **7**, 66 (2015).

[5] Martin P. J. Lavery, Mojtaba Mansour Abadi, Ralf Bauer, Gilberto Brambilla, Ling Cheng, Mitchell A. Cox, Angela Dudley, Andrew D. Ellis, Nicolas K. Fontaine, Anthony E.

Kelly, Christoph Marquardt, Selaelo Matlhane, Bienvenu Ndagano, Francesco Petruccione, Radan Slavík, Filippo Romanato, Carmelo Rosales-Guzmán, Filippus S. Roux, Kobus Roux, Jian Wang, and Andrew Forbes, Tackling Africa's digital divide, Nat. Photonics **12**, 249 (2018).

[6] Ningbo Zhao, Xiaoying Li, Guifang Li, and Joseph M. Kahn, Capacity limits of spatially multiplexed free-space communication, Nat. Photonics **9**, 822 (2015).

[7] Abderrahmen Trichili, Carmelo Rosales-Guzmán, Angela Dudley, Bienvenu Ndagano, Amine Ben Salem, Mourad Zghal, and Andrew Forbes, Optical communication beyond orbital angular momentum, Sci. Rep. **6**, 27674 (2016).

[8] Giovanni Milione, Martin P. J. Lavery, Hao Huang, Yongxiong Ren, Guodong Xie, Thien An Nguyen, Ebrahim Karimi, Lorenzo Marrucci, Daniel A. Nolan, Robert R. Alfano, and Alan E. Willner, Gbit/s mode division multiplexing over free space using vector modes and a q-plate mode (de)multiplexer, Opt. Lett. **40**, 1980 (2015).

[9] Mitchell A. Cox, Carmelo Rosales-Guzmán, Martin P. J. Lavery, Daniel J. Versfeld, and Andrew Forbes, On the resilience of scalar and vector vortex modes in turbulence, Opt. Express. **24**, 18105 (2016).

[10] Bienvenu Ndagano, Isaac Nape, Mitchell A. Cox, Carmelo Rosales-Guzman, and Andrew Forbes, Creation and detection of vector vortex modes for classical and quantum communication, J. Lightwave Technol. **36**, 292 (2018).

[11] Jaime A. Anguita, Mark A. Neifeld, and Bane V. Vasic, Turbulence-induced channel crosstalk in an orbital angular momentum-multiplexed free-space optical link, Appl. Opt. **47**, 2414 (2008).

[12] Jaime A. Anguita, Mark A. Neifeld, and Bane V. Vasic, Modeling channel interference in an orbital angular momentum-multiplexed laser link, Proc. SPIE **7464**, 74640U (2009).

[13] D. L. Fried, Statistics of a geometric representation of wavefront distortion, J. Opt. Soc. Am. **55**, 1427 (1965).

[14] S. Navidpour, Murat Uysal, and Mohsen Kavehrad, BER performance of free-space optical transmission with spatial diversity, IEEE Trans. Wirel. Commun. **6**, 2813 (2007).

[15] James H. Churnside and Steven F. Clifford, Log-normal Rician probability-density function of optical scintillations in the turbulent atmosphere, J. Opt. Soc. Am. A **4**, 1923 (1987).

[16] Mohammadreza A. Kashani, Murat Uysal, and Mohsen Kavehrad, A novel statistical channel model for turbulence-induced fading in free-space optical systems, J. Lightwave Technol. **33**, 2303 (2015).

[17] Junhe Zhou, Jinbang Zong, and Daoqiang Liu, Coupled mode theory for orbital angular momentum modes transmission in the presence of atmosphere turbulence, Opt. Express **23**, 31964 (2015).

[18] V. P. Aksenov, V. V. Kolosov, G. A. Filimonov, and C. E. Pogutsa, Orbital angular momentum of a laser beam in a turbulent medium: Preservation of the average value and variance of fluctuations, J. Opt. **18**, 054013 (2016).

[19] Yongxiong Ren, Zhe Wang, Guodong Xie, Long Li, Yinwen Cao, Cong Liu, Peicheng Liao, Yan Yan, Nisar Ahmed, Zhe Zhao, Asher Willner, Nima Ashrafi, Solyman Ashrafi, Roger D. Linquist, Robert Bock, Moshe Tur, Andreas F. Molisch, and Alan E. Willner, Free-space optical communications using orbital-angular-momentum multiplexing combined with MIMO-based spatial multiplexing, Opt. Lett. **40**, 4210 (2015).

[20] Li Zou, Le Wang, and Shengmei Zhao, Turbulence mitigation scheme based on spatial diversity in orbital-angular-momentum multiplexed system, Opt. Commun. **400**, 123 (2017).

[21] Murat Uysal, Carlo Capsoni, Zabih Ghassemlooy, Anthony Boucouvalas, and Eszter Udvary, eds., *Optical Wireless Communications, Signals and Communication Technology* (Springer International Publishing, Cham, 2016).

[22] John R. Barry, Edward A. Lee, and David G. Messerschmitt, *Digital Communication* (Springer, US, 2003), 3rd ed.

[23] John G. Proakis and Masoud. Salehi, *Digital Communications* (McGraw-Hill, 2008).

[24] D. Brennan, Linear diversity combining techniques, Proc. IRE **47**, 1075 (1959).

[25] Gilda Raoof Mehrpoor, Majid Safari, and Bernhard Schmauss, in 2015 4th International Workshop on Optical Wireless Communications (IWOW) (IEEE, 2015), p. 78.

[26] Bienvenu Ndagano, Nokwazi Mphuthi, Giovanni Milione, and Andrew Forbes, Comparing mode-crosstalk and mode-dependent loss of laterally displaced orbital angular momentum and Hermite-Gaussian modes for free-space optical communication, Opt. Lett. **42**, 4175 (2017).

[27] Anna T. O'Neil and Johannes Courtial, Mode transformations in terms of the constituent Hermite-Gaussian or Laguerre-Gaussian modes and the variable-phase mode converter, Opt. Commun. **181**, 35 (2000).

[28] L. Andrews, *Laser Beam Propagation through Random Media* (SPIE Press, Washington, DC, 2005), 2nd ed.

[29] Carmelo Rosales-Guzmán and Andrew Forbes, *How to Shape Light with Spatial Light Modulators* (SPIE Press, Washington, DC, 2017), p. 57.