Digital spiral-slit for bi-photon imaging

Melanie McLaren and Andrew Forbes

School of Physics, University of the Witwatersrand, Private Bag 3, Johannesburg 2050, South Africa
E-mail: andrew.forbes@wits.ac.za

Received 1 December 2016, revised 25 January 2017
Accepted for publication 6 February 2017
Published 21 March 2017

Abstract
Quantum ghost imaging using entangled photon pairs has become a popular field of investigation, highlighting the quantum correlation between the photon pairs. We introduce a technique using spatial light modulators encoded with digital holograms to recover both the amplitude and the phase of the digital object. Down-converted photon pairs are entangled in the orbital angular momentum basis, and are commonly measured using spiral phase holograms. Consequently, by encoding a spiral ring-slit hologram into the idler arm, and varying it radially we can simultaneously recover the phase and amplitude of the object in question. We demonstrate that a good correlation between the encoded field function and the reconstructed images exists.

Keywords: quantum imaging, orbital angular momentum, digital holography
(Some figures may appear in colour only in the online journal)

1. Introduction
Orbital angular momentum (OAM) of light, first recognized by Allen et al [1], has fueled numerous research fields [2], with the OAM eigenstates forming a complete, orthogonal and infinite-dimensional basis. Access to multiple OAM states allows for increased information capacity in optical communication [3, 4] and similarly provides additional data in imaging experiments. Digital spiral imaging was first demonstrated by Torner et al [5] in 2005, where they showed that information regarding an object could be obtained by analyzing the scattered OAM or spiral spectrum of a Gaussian beam incident on the object. This technique has been used to analyze canonical objects [6], detect rotating black holes [7] and study the diffraction of twisted light beams [8].

The demonstration of OAM conservation in the creation of entangled photons via spontaneous parametric down-conversion (SPDC) [9] encouraged investigations into quantum correlations of the photon pairs, otherwise known as ghost imaging. The concept of ghost imaging makes use of entangled photon pairs to reconstruct an image of an object that has only interacted with one of the photon pairs [10, 11]. Originally, the ghost-imaging technique combines information from two-photon detectors, a single pixel (bucket) scanning detector with no spatial resolution and a multipixel detector, to produce an amplitude image of the object. In this method, the bucket detector collects photons that have interacted with the object, while the multipixel detector collects those that have not encountered the object. However, ghost-imaging experiments using OAM quantum correlations have resulted in enhanced edge contrast of images [12, 13] and a demonstration of angular ghost diffraction [14, 15]. Recently, digital spiral imaging has been implemented in a ghost-imaging-type experiment, where the spiral spectrum of fractional helical phase elements were analyzed [16].

We expand on the concept of quantum digital spiral imaging by reconstructing spatially resolved samples. Using a radially-varying spiral ring-slit, we are able to not only recover the amplitude but also the phase information of the object. Our technique requires no mechanical moving parts.

2. Concept and theory
Digital spiral imaging takes advantage of the orthogonality of OAM modes such that an arbitrary optical field can be written in terms of the angular harmonics, \( \exp(i\ell \phi) \), where \( \ell \) is the azimuthal index. Using the Laguerre–Gaussian (LG) basis as the OAM basis, an unknown field \( u(r, \phi) \) can be
Figure 1. (a) Azimuthal ring restricting the azimuthal mode to a radial coordinate, (b) amplitude mask added to the restricted azimuthal mode, to yield (c) the spiral ring-slit of a particular thickness.

expressed as [17]:

\[ u(r, \phi) = \sum_{\ell} a_\ell(r) \exp(i\ell\phi), \]

(1)

where

\[ a_\ell(r) = \frac{1}{2\pi} \int_{0}^{2\pi} u(r, \phi) \exp(-i\ell\phi) d\phi. \]

(2)

Here \( a_\ell(r) \) denote the complex coefficients that depend on the radial coordinate and \( \ell \) is the azimuthal index. Thus the magnitude of the coefficients is found by an inner product calculation and an appropriate match filter. Experimentally, this is realized by measuring the signal at the origin of the Fourier plane of a lens placed after the modulation of the field by a transmission function, \( t_\ell(r, \phi) \). Mathematically, this is expressed as

\[ U_\ell(0) = \frac{\exp(ikf)}{i\ell f} \int_{0}^{\infty} \int_{0}^{2\pi} t_\ell(r, \phi) u(r, \phi) r dr d\phi, \]

(3)

where \( k \) is the wavenumber and \( f \) is the focal length of the Fourier lens. To satisfy equation (2), the transmission function must have an azimuthal phase variation opposite to the analyzed mode and must be able to select information as a function of \( r \). We thus chose to encode an annular ring of thickness \( \Delta R \), centered at \( r = R \) and defined as

\[ t_\ell(r, \phi) = \begin{cases} \exp(-i\ell\phi), & R - \Delta R/2 < r < R + \Delta R/2 \\ 0, & \text{otherwise} \end{cases} \]

(4)

An amplitude mask, defined as an array of pixels that varies from 0 to \( \pi \), added to the annular ring allows both the phase and amplitude of the field to be resolved. Figure 1 illustrates the transmission function used to measure the coefficients in equation (2). While this method allows unknown beams to be reconstructed in both phase and amplitude, it can also be applied to imaging, where an object is digitally generated with one hologram and then reconstructed using the ring-slit function.

More interestingly, this technique can be applied to ghost imaging, by generating entangled photons via SPDC, where a pump photon incident on a non-linear crystal produces a pair of photons. This photon pair is entangled in energy and momentum, including OAM entanglement, such that the two-photon state can be described by [9]

\[ |\Psi\rangle = \sum_{\ell} a_{\ell,-\ell} |\ell\rangle_A |\ell\rangle_B, \]

(5)

where \( |a_{\ell,-\ell}\rangle^2 \) is the probability of finding photon \( A \) in state \( |\ell\rangle \) and photon \( B \) in state \( |\ell\rangle \). As such, by placing an object in only one photon path and the spiral ring-slit in the other, the image can be reconstructed.

3. Results

Figure 2 illustrates the experimental setup implemented to generate entangled photon pairs and thereafter reconstruct the object through spiral imaging [18]. A type I BBO crystal, pumped by a mode-locked ultraviolet (355 nm wavelength), was used to generate collinear entangled photons via the SPDC process. To ensure only down-converted photons were transmitted, a bandpass filter centered on 710 nm was used to reflect the remaining pump light. Two lenses, \( L_1 \) (\( f = 200 \text{ mm} \)) and \( L_2 \) (\( f = 400 \text{ mm} \)), were used to image the front plane of the non-linear crystal to two separate spatial light modulators (SLMs); one to encode the object hologram and the other to encode the spiral ring-slit. The ring was encoded with a thickness of \( \Delta R = 0.08 \text{ mm} \) and was increased radially in steps of 0.016 mm for a fixed azimuthal value. Single-mode fibers (SMFs) were used to ensure only the fundamental Gaussian mode was detected. As such, a \( \times 250 \) demagnification telescope using \( L_3 \) (\( f = 500 \text{ mm} \)) and \( L_4 \) (\( f = 2 \text{ mm} \)) allowed the SLM planes to be imaged and coupled into the SMFs. The detection of single photons was recorded using avalanche photo-diodes (APDs) connected to a coincidence counter.

3.1. Back-projection

The propagation of classical light through the experimental system has been used to not only align single photon setups, but also as a means to experimentally simulate quantum entanglement results [19–21]. By replacing the detection system in arm A with a 710 nm diode laser and inserting a mirror in place of the BBO crystal, light can be propagated through the same setup in figure 2 but in reverse. Before simulating the spiral experiment, images of the encoded object were recorded on a CCD camera placed in the same plane as SLM B.

As an initial test of the technique, SLM A was encoded with a simple LG mode with the \( \ell = -1 \) fork hologram as the object. Figure 3(a) shows the intensity profile of the LG mode. A spiral ring-slit was encoded onto SLM B with an azimuthal value of \( \ell = +1 \). As the radial position of the ring-slit varied, the single count rates at detector B were recorded and plotted as shown in figure 3(b), where the purple dots represent the experimental data and the solid red line represents an LG fit. The insert shows the reconstructed object.

3.1.1. Down-converted results. Returning to the original configuration in figure 2, the technique was demonstrated with two different objects: an LG beam of \( \ell = 2 \) and a Bessel–Gaussian (BG) of \( \ell = 2 \). A fork hologram was initially encoded onto SLM A to represent the LG mode.
whereafter the method was repeated with a binary axicon [20] hologram used to represent the BG mode. The measured coincidence counts were plotted as a function of the radial position of the spiral ring-slit on SLM B. As shown in figure 4, both the amplitude and phase of these angular harmonics were successfully recovered.

These modes form a complete basis set, and as such superpositions can also be generated by summing together their transmission functions [22]. The mode superpositions were encoded onto SLM A, and by adjusting the spiral ring-slit radially, with a single azimuthal value encoded at a time, and recording the coincidence count rate, the fields were reconstructed. The recovered fields were correlated with the CCD images, shown as inserts in figure 5, to yield a correlation value of $\approx 0.9$.

Lastly, the digital technique was used to distinguish different phases within a single object. Figure 6(a) shows the object encoded onto SLM A, which consists of a circle with four different azimuthal indices each at different radii. That is, the $\ell = 0$ mode was positioned with an inner radius of 0 mm and an outer radius of 0.08 mm, while the $\ell = 5$ mode was positioned with an inner radius of 0.08 mm and outer radius of 0.16 mm. Similarly the $\ell = 3$ mode existed between 0.16 mm and 0.24 mm, and lastly, an azimuthal index of $\ell = 1$ was positioned between 0.24 mm and 0.32 mm. The spiral ring-slit was scanned across the radial coordinate four times, each scan with a different azimuthal value. By combining the measured coincidence count rate for each scan, the intensity field of the object was reconstructed, as shown in figure 6(b).

4. Discussion and conclusion

We have demonstrated a simple, digital ghost-imaging technique that allows both the phase and amplitude of an object to be reconstructed. The reconstructed intensity images closely resemble those recorded on a CCD camera. The spatial resolution could be improved by decreasing the width of the ring-slit, however at the cost of the coincidence count rate signal. Thus, a balance between resolution and efficiency is required. As such, the method was first illustrated in back-projection, where the count rate is significantly higher, allowing the ring thickness to be optimized.
Figure 4. (a) Radial profile of LG of $\ell = 2$, from a CCD image, (b) normalized coincidence count rate as a function of the radial position of the spiral-slit for an LG mode of $\ell = 2$. (c) Normalized intensity profile of a BG mode of $\ell = 2$, (d) normalized coincidence count rate as a function of the radial position of the spiral-slit for a BG mode of $\ell = 2$.

Figure 5. The superposition of an LG mode of (a) $\ell = -1$ and (b) $\ell = 2$ reconstructed down-converted counts images, similarly those of a BG mode of (d) $\ell = -1$ and (e) $\ell = 2$, resulting in petal structures (c) and (f), respectively. The CCD images of these modes are shown in the inserts.
Figure 6. (a) The phase object encoded on SLM A, with (b) its reconstructed intensity field. (c) The reconstructed phase pattern, including four different azimuthal values: (d) $\ell = 0$, (e) $\ell = 5$, (f) $\ell = 3$, and (g) $\ell = 1$.

Acknowledgments

MM and AF would like to thank Thandeka Mhlanga and Gareth Berry for their assistance in gathering data.

References