Hybrid quantum erasure scheme for channel disturbance characterization

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We demonstrate a simple projective measurement based on the quantum eraser concept that can be used to characterize the disturbances of any communication channel. Quantum erasers are commonly implemented as spatially separated path interferometric schemes. Here we exploit the advantages of redefining the which-path information in terms of spatial modes, replacing physical paths with abstract paths of orbital angular momentum (OAM). Remarkably, vector modes (natural modes of free-space and fibre) have a non-separable feature of spin-orbit coupled states, equivalent to the description of two independently marked paths. We explore the effects of fibre perturbations by probing a step-index optical fibre channel with a vector mode due to the increasing relevance in high-order spatial mode encoding of information for ultra-fast fibre communications.

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I. INTRODUCTION

The concept of which-way information has profound implications in the study of coherence of light, giving a different scope to the wave-particle historical duality. The first experimental demonstration of photons behaving as waves was performed by Young in 1804, in his famous double-slit interferometer [1] followed by modern variations of the same [2–6]. The double-slit experiment is a two-path interferometer in which a light source blocked by a screen with two slits is split into two new sources travelling along different paths. Upon propagation, these two new sources interfere with each other to produce an interference pattern of spatial fringes. Remarkably, interferometric phenomena are not restricted to two-path interferometers; it is also possible to observe interference of beams travelling along the same path.

The combination of the two degrees of freedom, orbital angular momentum (OAM) and polarization in a non-separable fashion, known as vector beams, allows to perform novel versions of the two-slit experiments. Here, the physical paths are replaced by two components of one degree of freedom, e.g., two values of OAM. Vector beams have gained significant amount of interest in a great variety of research fields at both the classical and the quantum levels. In particular, in the field of optical communications and quantum information, their high dimensional encoding capabilities have raised attention [7–10] due to their potential applications in free-space and optical fibers [11–13]. In quantum optics, photons entangled in OAM and polarization have been demonstrated to violate an analogous Bell-inequality [14], being able also to tune its entanglement or photon indistinguishability [15], similarly to the analogous version of a quantum eraser scheme using OAM and polarization [16]. Other degrees of freedom (DoF) can also be found to demonstrate this particular type of correlations, like in the case of generating entanglement between momentum and polarization in a single photon [17], or even using intense beams [18, 19].

The modern view of wave-particle duality has opened new research lines, for example in the development of novel measurement schemes, as the ones based on quantum non-demolition [20]. The traditional quantum eraser experiment [21–29], and its delayed choice versions are related to the complementarity principle [29–32] formulated by Bohr in 1928, which states that photons can behave indistinctly as particles or waves but cannot be observe as both simultaneously [33].

Importantly, the double-slit experiment and its modern variations allows to link the which-way information provided by the whole system with the interference pattern produced at the detection plane [34]. That is, the visibility of the interferometric pattern can be directly related to the properties of the system. For example, the decrease of quality in the interferometric measurement can be associated to the losses introduced by the system. This approach provides a useful tool for applications in optical communication in both free space and optical fibres [11–13], a hot topic nowadays due to the realization of a pending bandwidth “capacity crunch”.

Here we report on the comparison of a quantum eraser in free-space and a step-index fibre using the OAM and polarization degrees of freedom provided by vector modes. This work establishes the basis for a simple detection technique to quantify perturbations introduced by the environment, particularly in the communication channel between the source and the detection section. In our particular case, a fibre optic link is formed by many different channels, introducing some kind of perturbation depending on the degree of freedom that carries the encoded information. The results presented give a fast probing method to characterize the different channel’s perturbation, and would be useful for determining in an easy measurement what is feasible for a quantum communication channel down finer.

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II. CONCEPT

A. Revisiting the traditional which-way quantum eraser

In the traditional quantum eraser, a double slit is marked with orthogonal polarisers while an additional polariser is used to make judicial choices of obtaining the path information or erasing it. Quantitatively, this is associated with the complementarity inequality \[35–38],

\[ V^2 + D^2 \leq 1, \]

where \(D\) is the amount path information in the system while \(V\) is the visibility of interference fringes. Thus, gaining knowledge of path information \((D \neq 0)\), reduces the visibility of the fringes \((V < 1)\). To illustrate this, for example, consider a double slit marked with orthogonal linear polarisers given by

\[ |\Phi \rangle = \frac{1}{\sqrt{2}} (|H\rangle |s_1\rangle + |V\rangle |s_2\rangle), \]

being \(|s_1\rangle\) and \(|s_2\rangle\) the states upon traversing the independent paths (slits) \(s_1\) and \(s_2\), respectively. \(|H\rangle\) and \(|V\rangle\) represent the horizontal and vertical polarisers that mark the two slits (paths). Note that without the markers, the two paths are allowed to interfere, which leads to the trivial case of interference fringes appearing at the detection plane with \(D = 0\) (minimal path information) and \(V = 1\) (maximal fringe visibility) due to path indistinguishability (wave-like behaviour). On the contrary when the slits are marked, the probability distribution of the photons is \(|\langle \Phi |\Phi \rangle|^2 = \sum_{\ell} |\langle \psi_\ell |\psi_\ell \rangle|^2 / 2\), which signals the presence of path information in the system when projecting the polarisation of the system onto the \(|H\rangle\) or \(|V\rangle\) states. Thus \(D = 1\) (maximal path information) and \(V = 0\) (minimal fringe visibility) meaning that there is a full knowledge of the which-path information (particle like behaviour).

However, the interference fringes are recovered with a complimentary projection of the polarisation, in the diagonal basis \((|H\rangle \pm |V\rangle)\), which acts to remove the path information and hence erase it from the system. Again, \(D = 0\) (minimal path information) and \(V = 1\) (maximal fringe visibility) showing a mutually exclusivity between the two cases. Intriguingly, partial visibility and partial distinguishability are permitted, where the result cannot be explained exclusively by a wave-like or particle-like interaction although the inequality in Eq. 1 is maintained \[38].

B. Redefining the quantum eraser with spatial modes

Equation 2 represent a general state of a non-separable or entangled path and polarization degree of freedom (DoF) of a single photon, a trait of non-separable DoF of a photon \[17\]. Vector modes are a class of spatial modes with non-separable polarisation and OAM DoF with the following general form

\[ |\psi \rangle = \frac{1}{\sqrt{2}} (|R\rangle |\ell\rangle + e^{i\phi} |L\rangle |-\ell\rangle). \]

Here, \(e^{i\phi}\) is a relative phase, the states \(|\pm \ell\rangle\) are the OAM eigenstates with \(\ell\) representing the topological charge of the spatial field, characterised by a helical phase of \(e^{i\phi}\). \(|R\rangle\) and \(|L\rangle\) are the right and left circular polarisation states, respectively. In Eq. 3 the OAM eigenstate of the photon are marked with orthogonal circular polarisation states. Through polarisation control, the OAM information can be determined and erased. For example, by projecting the photon onto the polarisation state \(|R\rangle\) or \(|L\rangle\), the photon collapses onto the state \(|\ell\rangle\) or \(|-\ell\rangle\) \((D = 0, V = 1)\), respectively, where the spatial fields are azimuthal donut-like rings with opposite helicities. An example is illustrated in Fig.1(e) for \(\ell = \pm 10\). Analogously, the OAM eigenstates act as abstract paths in contrast to the double slit. The OAM modes can be interfered with a complimentary projection of the polarisation i.e \(|R\rangle \pm |L\rangle\), thus collapsing the spatial mode onto a superposition state, \(|\ell\rangle \pm |-\ell\rangle\), where the interference fringes appear in the azimuthal direction with a frequency proportional to \(2|\ell|\) (see Fig.1). Hence this erases the OAM information of the photon \((D = 0, V = 1)\).

Accordingly, the non-separability is exploited to demonstrate the quantum eraser with a single photon described by a vector mode. Interestingly, these spatial modes are natural modes of free space and fiber, the basic mediums of quantum information and communication.

C. Vector mode propagation in step index fibers

Step-index fibres have cylindrical symmetry and a refractive index with a step-like profile, as can be seen in Fig. 2. The full vector wave equation for a step-index fiber is given by

\[ \{\nabla_r^2 + n^2 k^2 \nabla_z\} u_\ell + \nabla\{u_\ell \cdot \nabla \ln(n^2)\} = \beta^2 u_\ell, \]

where \(k = 2\pi/\lambda\) is the wave vector, \(n\) is the index of refraction which has a radial dependence, \(u_\ell\) is the transverse component of the electric field while \(\beta\) is the propagation constant for each solution. The radial component of the fields are described as follows,

\[ u_\ell(r) = \begin{cases} J_{|\ell|}(\frac{\sigma_{r \ell}}{a})/J_{|\ell|}(\beta_{r \ell}) & r < a, \\ K_{|\ell|}(\frac{\sigma_{r \ell}}{a})/J_{|\ell|}(\sigma_{r \ell}) & r \geq a, \end{cases} \]

being \(a\) the fibre core radius, with the functions \(J_{|\ell|}\) and \(K_{|\ell|}\) representing the higher order Bessel and modified Bessel functions. The \(\beta_{r \ell}\) and \(\sigma_{r \ell}\) are the respective
FIG. 1. Intensity profile of vector modes described by Eq. (3), belonging to the $\ell = 1$ subspace. (a) is the TM$_{01}$ mode ($\zeta = 0$, $\ell = 1$) with field vectors pointing radially, (b) is the TE$_{01}$ mode ($\zeta = \pi$, $\ell = 1$) with azimuthal field lines, and the hybrid electric even (c) HE$_{21}^{\text{even}}$ ($\zeta = 0$, $\ell = -1$) and hybrid odd (d) HE$_{21}^{\text{odd}}$ ($\zeta = \pi$, $\ell = -1$) modes. (e) is an illustration of the appearance of azimuthal fringes when two OAM modes with a topological charge $\ell = \pm 10$ are in a superposition state. The fringes appear in the azimuthal direction with a frequency of $2|\ell|$. (f) is an illustration of the OAM quantum eraser where the non-separable spin-OAM coupling would be performed with a q-plate. The OAM would be distinguished and erased by a polarisation analyser were the spatial fringes are analysed by an azimuthal scanner of choice.

FIG. 2. A graphical illustration of the step-index fiber with a characteristic step-like index of refraction where the highest index of refraction is at the core and the lowest is found in the cladding.

propagation constants for the Bessel functions in the different regions of the fibre. Equation (5) is a consequence of the step-index fibres cylindrical symmetry and refractive index profile. The first four cylindrically symmetric higher-order vector solutions, which are nearly degenerate, take the form of Eq. (3). The first higher order transverse electric (TE$_{01}$), transverse magnetic (TM$_{01}$), hybrid electric odd (HE$_{21}^{\text{odd}}$) and hybrid electric even (HE$_{21}^{\text{even}}$) where the two indices represent the number of half-wave patterns across the width and the height of the waveguide, respectively.

In this paper, we consider the propagation of the TM$_{01}$ mode which is also known for its radial field profile and is defined by Eq. (3) (see Fig. 1 for intensity profiles) for $\ell = 1$ and $\zeta = 0$.

III. IMPLEMENTATION

To generate the spatial modes marked with orthogonal polarisation states we make use of a q-plate, a Pancharatnam-Berry phase element with a locally varying birefringence. A q-plate couples the polarization and OAM DoF of light according to the following rules:

$$|\ell\rangle|R\rangle \xrightarrow{\text{q-plate}} |\ell - 2q\rangle|L\rangle,$$
$$|\ell\rangle|L\rangle \xrightarrow{\text{q-plate}} |\ell + 2q\rangle|R\rangle,$$

being $|L\rangle$ and $|R\rangle$ the left and right circular polarisation states while $q$ represents the charge of the q-plate. The spin component of the photon is inverted with an addition in OAM of $2q$. For example, a photon (or photons) with a Gaussian transverse distribution given by $|R\rangle|0\rangle$, according the transformation rules, is converted to $|L\rangle|1\rangle$ if $q = 0.5$, through spin-orbit transfer of the q-plate. This corresponds to a scalar mode: a class of spatial modes with product states of polarisation and OAM DoF which are separable. In contrast, vector modes are superpositions of these modes where on the contrary, the OAM and polarisation entities are non-separable. These states are generated by preparing linearly polarised photons, for example $\frac{1}{\sqrt{2}}(|R\rangle|0\rangle + |L\rangle|0\rangle)$, set by using polarisation optical elements. Accordingly, the state of the photon upon traversing the q-plate is given by,

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|R\rangle|\ell\rangle + |L\rangle|-\ell\rangle)$$

where $\ell = 2q$. Equation (8) reminds us of the quantum state represented by two paths distinguished by orthogonal polarisation markers (see Eq. 2). We carry out the required projections for the quantum eraser on a photon encoded with the state presented in Eq. (8) through polarisation control followed by a pattern sensitive scanning technique. To achieve this, we firstly convert the polarisation of the photon from the circular to linear basis with a $\frac{\lambda}{4}$ wave-plate oriented at 45° with respect to the horizontal. Secondly, a polarisation analyser orientated at an
angle $\alpha$ (with respect to the horizontal), will project onto the following target state

$$|\alpha\rangle = \cos(\alpha) |H\rangle + \sin(\alpha) |V\rangle,$$  \hspace{1cm} (9)

thus allowing the “path” to evolve from marked to unmarked by a judicious choice of $\alpha$. Next, the visibility of spatial fringes needs to be detected, which may easily be done with scanning detectors (or more expensive camera-based systems). We instead make use of scanning holograms and a fixed detector as our pattern sensitive detector \[41\]. We create sector states from superpositions of OAM with a relative intermodal phase of $\theta$,

$$|\theta\rangle = \left( |\ell\rangle + e^{2\theta} |\ell\rangle \right).$$ \hspace{1cm} (10)

The phase structure of $|\theta\rangle$ is azimuthally periodic, and allows a measurement of the path (OAM) interference, analogous to detecting OAM entanglement with Bell-like measurements [12]–[45]. Thus the fringe pattern (or lack thereof) can be detected by scanning through $\theta$.

The normalized probability of detection given the two projections is

$$P(\alpha, \theta) \propto |\langle \theta | \alpha \rangle |^2$$

$$= \frac{1}{2} \left( 1 + \sin(2\alpha) \cos(2\theta + \delta) \right).$$ \hspace{1cm} (11)

Here $P(\alpha, \theta)$ is equivalent to the theoretical photon counts. When the polariser is orientated at $\alpha = 0^\circ$, which corresponds to the $|H\rangle$ polarization state, the probability distribution with respect to $\theta$ is a constant since the path is marked. Conversely, for $\alpha = \pm 45^\circ$ which corresponds to complimentary polarization projections on $|D\rangle$ or $|A\rangle$, then $P(\alpha = \pm 45^\circ, \theta) \propto 1 \pm \cos(\theta + \delta)$ and hence the sinusoidal dependence is an indication of an interference pattern emerging from a superposition of the OAM abstract paths. Therefore the which-path (OAM) information has been erased. The fringe visibility is given by

$$V = \frac{P_{\text{min}} + P_{\text{max}}}{P_{\text{max}} + P_{\text{min}}},$$ \hspace{1cm} (12)

where $P_{\text{min}}$ and $P_{\text{max}}$ are the maximum and minimum photon counts from rotating the azimuthal spatial mode analyser (SLM). The theoretical visibility of the spatial fringes with respect to the angle of the polariser $\alpha$ is

$$V = |\sin(2\alpha)|.$$

IV. EXPERIMENTAL SET-UP

We illustrate the concept for the both generation and detection of our intra-particle quantum eraser with vector modes in free-space and step-index fibre with the aid of Fig.\(3\). In our experiment, we made use of an attenuated laser as the single-photon source. The incident beam was horizontally polarized in order to be able to generate our vector mode, by means of correlating the polarization with the OAM degrees of freedom using a $q$-plate ($q = 0.5$) \[39, 40\]. The resulting state after the $q$-plate is given by Eq. (8), and is the one coupled into the step index fibre as seen schematically in Fig.\(2\). In order to give a proper analysis of the hybrid state disturbance introduced by the step-index fibre, we first performed the projective measurement to the free-space transmitted vector mode to have a reference curves for the best case scenario.

The detection of the hybrid mode of Eq. (8) was carried out by projecting first the polarization degree of freedom with a set of QWP, HWP and polariser, followed by binary phase masks, which were encoded on a phase-only SLM (Holoeye PLUTO) to project onto the particular spatial mode of the photon. This was done for $\alpha = [0^\circ, 45^\circ]$, while scanning holograms through $\theta = [0^\circ, 360^\circ]$. The projected photons were collected using a single mode fibre and detected with avalanche photo-diodes (Perkin-Elmer).

The key part of the experiment consists in coupling the vector mode generated after the $q$-plate within the step-index fibre core, and give an intuitive result based in the quantum eraser experiment, showing how much the hybrid state is affected by the disturbance introduced by the fibre. An objective $(20\times)$ was used to improve the coupling of the probing mode into the fibre, being able to measure its disturbance by performing the projective measurement just described.

V. RESULTS

In the work presented here we explore the analogy between the path and the OAM degrees of freedom in a typical which-way information experiment. In this par-
FIG. 4. Experimental results for the free space erasure showing that the “which-OAM” information is distinguishable for $\alpha = 0^\circ$ with visibility $V = 0.002 \pm 0.06$ or (b) indistinguishable for $\alpha = 45^\circ$ with $V = 0.99 \pm 0.02$. In (b) the intermediate cases are investigated by varying the polarization projections from $\alpha = 0^\circ$ to $45^\circ$ were the experimental visibility increases as the OAM information is depleted. A sign of complementarity.

FIG. 5. Experimental erasure results using the step-index fibre channel. The extreme case for $\alpha = 0^\circ$ with a visibility $V = 0.17 \pm 0.02$ and for $\alpha = 45^\circ$ with a visibility of $V = 0.98 \pm 0.04$.

particular case, the path information is encoded into a OAM mode, and thanks to its correlation with the polarization generated in the $q$-plate, the which-way information can be erased by projecting the hybrid state in the diagonal polarization from Eq. (9). Figure 4 show the results for the best case scenario, when the vector mode, used later to probe the step-index fibre, is just transmitted in free-space to give a reference curve.

As can be seen in Fig. 4(a), the visibility of the spatial fringes that appear after projecting with the spatial superposition described in Eq. (10), can be maximized or minimized depending on the polarization projection $\alpha$ value from Eq. (9). Being $\alpha = 45^\circ$ the projection value with maximum visibility, corresponding to the which-way information maximum erasure. And $\alpha = 0^\circ$ being the case where the visibility is minimized due to the spatial superposition projection of a single OAM mode. That is to say that low visibility for the $\alpha = 0^\circ$ case, corresponds of obtaining low spatial cross-talk between OAM modes within the given communication channel. Figure 4(b) shows the complete range of visibilities from the spatial fringes, with respect to the polarization projection $\alpha$ values. The maximum visibility obtained in the free-space configuration was $V = 0.99 \pm 0.01$, and $V = 0.02 \pm 0.06$ the minimum.

In Fig. 5 we see from the spatial fringe curves that the fibre affects the hybrid transmitted state. From the visibility of the $\alpha = 45^\circ$ case we can deduce the quality of erasing of the which-way information after the photon has travelled through the disturbing quantum channel. The maximum visibility obtained was $V = 0.98 \pm 0.04$, which contrasts well with the reference free-space results, meaning that the cross-talk affects the spatial modes in a symmetric manner. Thus, the spatial superposition is maintained as far as the good coupling into the step-index fibre core is preserved. In the contrary, when analysing the results of $\alpha = 0^\circ$ in Fig. 5, the minimum visibility curve ($V = 0.17 \pm 0.02$) is poorer than the reference free-space curve from Fig. 4(b), implying that the spatial state projected after the step-index fibre is no longer a single OAM mode due to the cross-talk within the core, increasing the spatial superposition between OAM modes and increasing also the spatial fringes visibility correspondingly. As can be observed in the curves of Fig. 5, the mean number of photons detected in the case of $\alpha = 0^\circ$ it is not the half of the normalized mean number of photons detected in the case of $\alpha = 45^\circ$. This is because of alignment complications due to the laser instability and step-index miss-coupling after a few minutes. Further, the shape of the spatial fringes is perturbed due to the changes in the polarization and spatial DoF within the fibre. These visibilities and curves provide practical
limits on what may be achieved in a quantum communication link down a fibre, and serve as a fast "pretest" to a full quantum key distribution or quantum state transfer down fibre.

The complete analogy between path information and spatial fringe visibility is essential to our quantum eraser scheme concept. By defining the two distinct paths using the OAM DoF, we have shown that through polarization-OAM hybrid state, it is possible to describe a simple projection scheme measurement capable of showing the total amount of disturbance present in a particular communication channel.

Taking into account that any degree of freedom may be used to perform such a simple projective measurement, polarization-OAM hybrid state is an attractive choice due to the possibility to explore the impact of spatial dimensionality in optical communication links. That is because both degrees of freedom can be compensated after or before a particular disturbance in a communication channel by a unitary transformation, needing only a simple and fast detection scheme to implement a feedback measurement to compensate. Previous works have shown that the non-separability of vector modes [46] can be advantageous in measuring techniques for quantum entanglement to quantitatively determine their mode quality in free-space [47] and [13] fibre. Here we have performed a new channel measurement in fibre.

In conclusion, we have shown the complete analogy in the which-way information concept, when using a path encoding approach or using instead a spatial DoF such as the OAM of a single photon. We have also derived a simple and fast projective detection scheme to measure the total amount of the environmental effect on a polarization-OAM hybrid state, independently of the quantum channel used. We have focused our efforts in studying the step-index optical fibre channel due to the increasing relevance in high-order spatial encoding of information for ultra-fast fibre communications.

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