Optical interference with digital holograms

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In 1804, Thomas Young reported the observation of fringes in the intensity of light, and attributed it to the concept of interference between coherent sources. In this paper, we revisit this famous experiment and show how it can easily be demonstrated with digital holography. We look closely at the concept of interference with light and ask, “fringes in what?” We then show that depending on how light interferes, fringe patterns in observables other than intensity can be seen. We explain this conceptually and demonstrate it experimentally. We provide a holistic approach to the topic, aided by modern laboratory practices for a straightforward demonstration of the underlying physics. © 2016 American Association of Physics Teachers. [http://dx.doi.org/10.1119/1.4948604]

I. INTRODUCTION

What is light? This is an age old question that people have been seeking the answer to since the times of the ancient Greek philosophers. Along with it has come the question of whether light is a wave or a particle. By the 18th century, the known properties of light had been explained separately by the two seemingly conflicting views, thus showing the validity of both. Despite this, the particle view was more widely supported, as it had been advocated by Newton. In the early 19th century, Thomas Young presented his landmark experiment showing the interference of light, which finally turned the tides in favor of the wave nature of light.

On November 12, 1801, Thomas Young, without conclusive proof, put forward to the Royal Society the idea that light propagates as a wave through the luminiferous Ether, in the same way that sound travels through the air. He used passages of Newton’s work to show that Newton was actually first to put forward this theory, and stated that Newton’s opinions, “varied less from this theory than is now universally supposed.” In this theory, something becomes luminous by exciting undulations in the Ether. Different colors are perceived by the different frequencies of vibration that these undulations cause in the cells of the retina. In 1804, Young presented his famous double-slit experiment to The Royal Society, thus proving the wave nature of light. He demonstrated that when light from a single coherent source is made to pass through two apertures, fringes in intensity (light and dark patches) are seen on a screen some distance away where the light from the apertures recombines. Young postulated that these fringes were a result of interference between light waves propagating from the different apertures, in the same way waves in water interfere (see Fig. 1). He substantiates this claim by showing that the fringes correspond to positions where the path difference between the two portions of light are in arithmetical progression, as would be expected for wave-like interference. Since then, the concept of interference has found many applications, while its basic foundations remain an exciting field.

There is a general misconception that interference fringes are always found in intensity. This is enforced by the paradoxical nature of Young’s hypothesis: that light plus light can result in darkness. In this work, we revisit the concept of interference by looking for fringes not only in intensity but also spin angular momentum (polarization) and orbital angular momentum (OAM). We explain the phenomenon in terms of position and momentum space representation: fringes in linear position are associated with the linear momentum of light (lateral skewing of the rays), whereas fringes in angular position are associated with the angular momentum of light (angular skewing of the rays). We explain how the Fourier relation between position and momentum space gives rise to the well known interference properties of such beams.

Furthermore, we apply modern digital techniques in our study to observe the aforementioned interference effects in the laboratory.

The remainder of this manuscript is organized as follows. In Sec. II, Young’s theory of interference is reviewed in terms of phase differences for various fringe geometries. Section III introduces the working principle of the Spatial Light Modulator (SLM); we describe its basic operation as well as how digital holography can be implemented for our goals. Polarization fringes are presented in Sec. IV together with their corresponding experimental results. In Sec. V, we use the position and momentum space representation to explain interference phenomena and fringes in OAM. Finally, the discussion is presented in Sec. VI.

II. FRINGES IN INTENSITY

In Young’s passages, the concept of interference is described as: “The law is, that when two equal portions of light, in circumstances exactly similar, have been separated and coincide again, in nearly the same direction, they will
either co-operate, or destroy each other, accordingly as the difference of the times, occupied in their separate paths, is an even or an odd multiple of a certain half interval, which is different for the different colors, but constant for the same kind of light
cite{6} (see Figs. 1 and 2). Mathematically, this can be explained as follows. Let’s consider two monochromatic waves polarized in the same plane

\[ E_1 = A \exp(-i\varphi_1) \exp(iot) \]  

(1)

and

\[ E_2 = A \exp(-i\varphi_2) \exp(iot), \]  

(2)

where A represents an amplitude factor while \( \varphi_1 \) and \( \varphi_2 \) are the phases corresponding to the fields. We can express its coherent superposition as

\[ E = E_1 + E_2, \]

(3)

\[ = A \exp(iot)[\exp(-i\varphi_1) + \exp(-i\varphi_2)], \]

(4)

and by computing the intensity we get

\[ I = |E|^2 = 4A^2 \cos^2 \left( \frac{\varphi_1 - \varphi_2}{2} \right). \]

(5)

From Eq. (5), we can obtain the condition to observe constructive or destructive interference. The former is achieved if \( \varphi_1 - \varphi_2 = 2n\pi \), and the latter occurs if \( \varphi_1 - \varphi_2 = (2n-1)\pi \), for \( n \in \mathbb{Z} \). We can understand the quantity \( \varphi_1 - \varphi_2 \) as the path length difference described by Young. For instance, by imposing \( \varphi_1 = k_1x \) and \( \varphi_2 = k_2x \), the constructive interference will be located at

\[ x = \frac{2n\pi}{k_1 - k_2}, \]

(6)

where \( x \) represents the horizontal spatial coordinate, \( k_1, k_2 \) are related to the directions of the respective fields, and \( n \) is some integer. This linear relation was reported by Young through his double slit apparatus.

However, we note that there is no coordinate restriction in Eq. (5), and indeed we can assign appropriate values of \( \varphi_1 \) and \( \varphi_2 \) to achieve intensity fringes in any geometry. To get fringes in the azimuthal direction, we use beams whose phase dependence is of the form \( \exp(i\theta) \) known as vortex beams. These optical fields belong to the class of helical modes and possess an OAM content of \( \ell \hbar \) per photon.

\cite{7,8,9} For example, azimuthal fringes are constructed if we let \( \varphi_1 = \ell \theta \) and \( \varphi_2 = -\ell \theta \). Under these conditions, the constructive interference will be at

\[ \theta = \frac{n\pi}{\ell}, \]

(7)

where \( \theta \) is the azimuthal angle and \( \ell \) is an integer known as topological charge. In this case, the phase difference \( \varphi_1 - \varphi_2 \) represents a superposition of beams with OAM content.

To see this, note that from Eq. (5) we have that

\[ \ell \propto \cos^2(\ell\theta), \]

which represents a set of peaks and troughs (fringes) around the azimuth, with precisely \( 2\ell \) peaks. These fringes manifest as petals of light.

Following this, we can expand the analysis to obtain radial fringes. To illustrate, let \( \varphi_1 = k_\rho \rho \) and \( \varphi_2 = 0 \) whose constructive interference will have the form

\[ \rho = \frac{2n\pi}{k_\rho}, \]

(8)

where \( \rho = \sqrt{x^2 + y^2} \) is the radial coordinate, and consequently Eq. (8) shows radial fringes (concentric circles).

One can intuitively conceive of how to achieve this in a simple manner. Since pinholes at \( x \) and \( -x \) give fringes in the \( x \)-plane, and pinholes at \( y \) and \( -y \) give fringes in the \( y \)-plane, one can imagine continuing with this trend and

Fig. 1. Original diagram by Young to explain interference in wave phenomena (Ref. 5).

Fig. 2. Schematic representation of Young’s experiment. Constructive interference occurs when the waves from the two apertures meet in phase and destructive interference occurs where they meet out of phase. The phase of both waves at the apertures is the same because they are illuminated by a single source. The path difference \( \delta \) between the light from the different apertures meeting at some point on the screen determines the nature of the interference (constructive, destructive, or somewhere in between).
creating pinholes for all positions, or tracing out a ring (see Fig. 3). Thus, radial fringes can be understood as the superposition of infinitely many point fields arranged in a ring. Mathematically, we can state this as

\[ E = \sum_{n=0}^{N} E_n, \]

\[ = \sum_{n=0}^{N} A \exp(i\varphi_n), \quad \text{(10)} \]

where \( E_n \) is the \( n \)th field of the superposition and \( \varphi_n = k \cdot r \); is the phase corresponding to each field. Similarly, one can prepare a superposition of plane waves arranged in a cone. For this reason, we write \( k \) and \( r \) as

\[ k = k \sin \theta \cos \phi \hat{x} + k \sin \theta \sin \phi \hat{y} + k \cos \theta \hat{z}, \quad \text{(11)} \]

and

\[ r = \rho \cos \alpha \hat{x} + \rho \sin \alpha \hat{y} + z \hat{z}, \quad \text{(12)} \]

where \( r = r(\rho, \alpha, z) \) is the position in polar coordinates, and we have expressed \( k \) in spherical coordinates for a fixed value \( \theta \) and \( \phi \in [0, 2\pi] \). By employing the conditions in Eq. (10), we get

\[ E = A \sum_{n=0}^{N} \exp \left[ i(k \sin \theta \cos \phi_n \rho \cos \alpha + k \sin \theta \sin \phi_n \rho \sin \alpha + k \cos \theta z) \right], \quad \text{(13)} \]

or

\[ E = A \sum_{n=0}^{N} \exp \left[ i(k \sin \theta \rho \cos(\phi_n - \alpha) + k \cos \theta z) \right]. \quad \text{(14)} \]

The last equation can be understood as follows: each point in a ring possesses a radial phase dependence propagating towards the \( z \)-axis, which implies oscillations in the radial direction. The limit case occurs when \( N \to \infty \) and leads to

\[ E = \lim_{N \to \infty} A \sum_{n=0}^{N} \exp \left[ i(k \sin \theta \rho \cos(\phi_n - \alpha) + k \cos \theta z) \right], \quad \text{(15)} \]

Finally, we can rewrite Eq. (16) as

\[ E = A \exp(ik \cos \theta z) \int_{0}^{2\pi} \exp \left[ i(k \sin \theta \rho \cos(\phi - \alpha)) \right] d\phi. \quad \text{(16)} \]

whose intensity pattern shows concentric rings that matches with the desired radial fringes. In fact, this field is simply the well known Bessel beam, famous for its non-diffracting and self-healing properties. They were first proposed by Durnin,11,12 where indeed he created them using a ring slit aperture (see Fig. 4). Today they may also be created digitally or with an axicon lens.

III. DIGITALLY CREATING FRINGES

Before we look into the details of the interference experiments, a brief overview of the phase-only Spatial Light Modulator (SLM) will be given. An SLM consists of a liquid crystal display (LCD) that is able to electrically control the birefringence. By computing gray-level images and encoding them onto the device, we can modulate the phase of an incident beam to produce a beam possessing exceptional characteristics. The nature of the SLM allows us only to encode information on the polarization component parallel to the liquid crystal molecules. To remove the undiffracted component from the desired, diffracted component, a grating is placed over the hologram, separating the undiffracted zero-order and the diffracted first-order to two independent lateral positions. By the same means, one is able to modulate amplitude by applying a phase grating or a checkerboard pattern. Note that while we have used a phase-only SLM, SLMs do exist that modulate the amplitude and phase, or only amplitude. Such devices can also be used in phase-only mode,13 for example, by use of two appropriately aligned polarizers, or in a binary phase configuration. In general, the device may have a coupled amplitude and phase response and the reader is encouraged to correct this prior to implementing the holograms discussed here. For further details on the functioning of the SLM, please refer to Refs. 14–17.

Fig. 3. Diagram of wave vectors: (a) conic geometry, (b) side view (note that \( k_z = k_x \hat{x} + k_y \hat{y} \)), and (c) view along \( z \)-axis.

Fig. 4. Bessel beam generation through annular aperture; the beams propagate in a conical manner.
The experimental setup to create the intensity fringe patterns is depicted in Fig. 5. We directed and expanded a linearly polarized HeNe laser source ($\lambda = 632.8$ nm) onto an SLM (HoloEye PLUTO VIS SLM with 1920 x 1080 pixels of pitch 8 $\mu$m and calibrated for a 2$\pi$ phase shift at $\lambda = 632.8$ nm). In Fig. 6(b), we show the digital slits in the SLM encoded with a blazed grating. After the reflection from the SLM, we filtered out the undesired light keeping only the first order. Lenses L3 and L4 (Fig. 5) were used for the filtering of the desired order while L5 allowed the far-field of the created pattern to be observed. Finally, the intensity distribution was captured with a CMOS sensor (Firefly FMVU-13S2C). To create fringes in the azimuthal direction, we repeated the same procedure described above, but the digital apertures were changed by the hologram of a superposition of two OAM modes with opposite helicity, as shown in Fig. 6(d). Following our earlier discussion, the transmission function follows a $\cos^2(\phi)$ dependence, which can be represented as an amplitude hologram or as a phase-only binary approximation, as shown here. In this example, two peaks and two troughs are created, corresponding to OAM modes with $\ell = 1$ and $\ell = -1$. For the radial case we encoded from 2 to 50 circular apertures equally spaced and arranged in a ring, until the full ring was created, as shown in Fig. 6(f). The results for linear, azimuthal, and radial fringes are shown in Fig. 7, as well as in movie format. The azimuthal examples are for $\ell = \pm 1$, $\pm 3$, and $\pm 5$ (2 petals, 6 petals, and 10 petals, respectively). In the radial case, as the number of apertures making up the ring increases, so the visibility of the radial fringe pattern increases, as seen in panels (c), (f), and (i) in Fig. 7.

IV. FRINGES IN POLARIZATION

It is commonly believed that adding two orthogonally polarized fields leads to no interference effects, this is because while the addition of two fields of the same polarization can lead to zero, $E + (-E) = 0$, the addition of orthogonal polarizations cannot lead to zero, $E_1^2 + E_2^2 \neq 0$, except in the trivial case of no light at all. But this is not completely true; it is possible to observe fringes, but in its polarization rather than intensity. Wood noticed this behavior in thin film...
defined by the time variation of the electric field direction respectively. After carrying out some algebra, we find25

\[ \mathbf{e}_1 = \frac{1}{\sqrt{2}} \exp(i\varphi_1)\mathbf{x} + \frac{1}{\sqrt{2}} \exp(i\varphi_2)\mathbf{y}, \]

and

\[ \mathbf{e}_2 = \frac{1}{\sqrt{2}} \exp(-i\varphi_2)\mathbf{x} - \frac{1}{\sqrt{2}} \exp(-i\varphi_1)\mathbf{y}, \]

where \( \{\mathbf{e}_1, \mathbf{e}_2\} \) represent the elliptical polarization basis and \( \{\mathbf{x}, \mathbf{y}\} \) the linear horizontal and vertical polarization basis. To generate polarization fringes, let’s consider the following orthogonal superposition

\[
E(x, y) = \frac{1}{\sqrt{2}} A_1(x, y) \exp(i\varphi_1)\mathbf{x} + \frac{1}{\sqrt{2}} A_2(x, y) \exp(i\varphi_2)\mathbf{y},
\]

where \( A_1 = A_2 = A \) represent the amplitudes of the components. Now, we rewrite Eq. (23) in the elliptical basis

\[
E(x, y) = \sqrt{2} A(x, y) \hat{e}_1,
\]

where \( \hat{e}_1 = (\varphi_1, \varphi_2) \). Equation (24) tells us that the general state of the constructed field is elliptical; moreover, one can choose a particular \( \varphi_1, \varphi_2 \) with a spatial dependence such that a polarization varying field is created. To easily observe the polarization pattern, we can project into one of the elliptical polarization states. For simplicity, we choose the diagonal and anti-diagonal polarizations that can be observed with a linear polarizer at 45° and -45°, respectively, at the plane of the CCD detector (see Fig. 9).

If we want linear fringes along the \( x \)-axis in polarization, we choose \( \varphi_1 = k_1 x \) and \( \varphi_2 = k_2 x \). Physically, this means an orthogonal superposition of plane waves with different \( k \) values. The constructive interference condition of the diagonal component reads

\[
x = \frac{2\pi}{k_1 - k_2},
\]

where \( \delta = \varphi_1 - \varphi_2 \). Equation (20) is known as the polarization ellipse and represents the general polarization state. By changing the value of \( \delta \) we go from linear polarization to circular polarization states. Because \( \delta \) is dependent on the phase difference \( \varphi_1 - \varphi_2 \), one can construct a polarization varying map. Notice that \( \varphi_1 \) and \( \varphi_2 \) can possess a spatial dependence, and for this reason we can use the results from Sec. II.

The general polarization state is an ellipse, whose basis can be written as25

\[
\mathbf{e}_1 = \frac{1}{\sqrt{2}} \exp(i\varphi_1)\mathbf{x} + \frac{1}{\sqrt{2}} \exp(i\varphi_2)\mathbf{y},
\]

and

\[
\mathbf{e}_2 = \frac{1}{\sqrt{2}} \exp(-i\varphi_2)\mathbf{x} - \frac{1}{\sqrt{2}} \exp(-i\varphi_1)\mathbf{y},
\]

where \( \{\mathbf{e}_1, \mathbf{e}_2\} \) represent the elliptical polarization states and \( \{\mathbf{x}, \mathbf{y}\} \) the linear horizontal and vertical polarization basis.

The polarization describes light’s vector nature and is defined by the time variation of the electric field direction \( \mathbf{E}(\mathbf{r}, t) \) at a point \( \mathbf{r} \). We can express a vector field as

\[
\mathbf{E} = E_{0x} \cos(kz - \omega t + \varphi_1)\mathbf{x} + E_{0y} \cos(kz - \omega t + \varphi_2)\mathbf{y},
\]

where \( z \) is the propagation direction, \( k = 2\pi/\lambda \) is the wave vector, \( \omega \) is the frequency, \( t \) is time, and \( \varphi_1 \) and \( \varphi_2 \) are the phases associated with the horizontal and vertical components, respectively. After carrying out some algebra, we find25

\[
\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2 \frac{E_x}{E_{0x}} \frac{E_y}{E_{0y}} \cos \delta = \sin^2 \delta,
\]

\[
\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2 \frac{E_x}{E_{0x}} \frac{E_y}{E_{0y}} \cos \delta = \sin^2 \delta,
\]

\[
\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2 \frac{E_x}{E_{0x}} \frac{E_y}{E_{0y}} \cos \delta = \sin^2 \delta,
\]
which represents a family of lines along the \(x\)-axis. To generate linear polarization fringes, we repeated the digital Young experiment described in Sec. III, with an extra half-wave plate in the path of one aperture (see Fig. 9). The results are shown in Fig. 10, which shows (a) the intensity, where the interference phenomenon cannot be observed, as well as the (c) diagonal and (d) antidiagonal projections, where the fringe pattern is clear. We also present the theoretical polarization map in Fig. 10(b).

To obtain polarization fringes in the radial direction, we choose \(u_1 = k q_0\) and \(u_2 = 0\), where \(q_0 = \sqrt{x^2 + y^2}\) is the previously defined radial coordinate. Besides the Bessel case illustrated for intensity fringes, this realization can be implemented by the orthogonal superposition of a spherical wave and a plane wave propagating in the \(z\)-direction. By looking for constructive interference in the diagonal component we obtain

\[
\rho = \frac{2n\pi}{k} \tag{26}
\]

Notice that Eq. (26) represents a family of concentric circles. To obtain polarization fringes in the radial direction, we choose \(u_1 = k q_0\) and \(u_2 = 0\), where \(q_0 = \sqrt{x^2 + y^2}\) is the previously defined radial coordinate. Besides the Bessel case illustrated for intensity fringes, this realization can be implemented by the orthogonal superposition of a spherical wave and a plane wave propagating in the \(z\)-direction. By looking for constructive interference in the diagonal component we obtain

\[
\rho = \frac{2n\pi}{k} \tag{26}
\]

To observe radial fringes in polarization, we built the setup shown in Fig. 11. We directed a linearly polarized HeNe laser source (\(\lambda = 632.8\) nm), whose polarization axis is orientated at 45°, onto the first half of an SLM. In this way the horizontal component sees the hologram encoded on the SLM, while the vertical component is just reflected. The addition of a quarter-wave plate at 45° in front of a mirror rotates the polarization of the field, and reflects the beam back to the second half of the SLM. In the first reflection on the SLM one component is affected, while the second pass encodes the information in the orthogonal component. By doing so, we obtain an orthogonal superposition with arbitrary phases. The first half encodes the phase of a lens, while the second half just reflects the beam [see Fig. 12(a)]. Hence, we constructed the orthogonal superposition of a Gaussian beam and a light field with spherical phase evolution. In Fig. 13, we show (a) the intensity of the field, where the interference phenomenon cannot be observed, as well as the (c)
diagonal and (d) antidiagonal projections, where the fringe pattern is clear. The theoretical polarization map is shown in panel (b).

Finally, to generate polarization fringes in the azimuthal direction we select \( \varphi_1 = l\theta \) and \( \varphi_2 = -l\theta \), where \( \theta = \tan^{-1}(y/x) \). This can be implemented by the orthogonal superposition of vortex beams. The constructive interference in the diagonal component has the form

\[
\theta = \frac{n\pi}{l},
\]

where the above equation corresponds to a family of lines crossing the origin with slope \( \tan \left( \frac{n\pi}{l} \right) \). To experimentally address azimuthal fringes in polarization, we again use the setup shown in Fig. 11 that was used to generate radial fringes. The difference lies in the encoded holograms. In this case, the first half encoded the phase of an OAM mode, while the second half encoded the same OAM with opposite handedness [see Fig. 12(b)]. In this way, we constructed an orthogonal superposition of two vortex beams with opposite sign. The results are shown in Fig. 14, where we observe the existence of the interference phenomena through its spatially varying polarization. A similar procedure can be repeated to explore other forms of \( \varphi_1 \) and \( \varphi_2 \) and achieve more exotic polarization fringe patterns.

V. FRINGS IN ORBITAL ANGULAR MOMENTUM

Besides the mathematical treatment previously described, one can also rely on the position and momentum space representation. Both quantities are known to be related through the Fourier transformation, which in physical optics corresponds to the far-field propagation or the thin lens effect at the focal distance. In other words, interference phenomena can be fully explained by means of the Fourier transform. Furthermore, the subject of interference is a manifestation of Heisenberg’s uncertainty principle, most commonly understood as uncertainty between position and linear momentum, but just as applicable in terms of angular position and orbital angular momentum.

The pair of transformations can be defined as follows:

\[
\hat{\psi}(k) = \frac{1}{\sqrt{2\pi}} \int \psi(r) \exp(-i k \cdot r) \, dr \tag{28}
\]

and

\[
\psi(r) = \frac{1}{\sqrt{2\pi}} \int \hat{\psi}(k) \exp(i k \cdot r) \, dk, \tag{29}
\]

where \( \psi \) is the electric field, \( r \) is position, and \( k \) is momentum. We can easily outline Young’s experiment by means of the position and linear momentum representation. Let us impose the physical conditions of the Young experiment through the following expression

\[
\psi(r) = \delta(r - r_0) + \delta(r + r_0), \tag{30}
\]

where \( \delta \) is the Dirac delta-function, and \( r_0 \) represents an arbitrary location. By using Eq. (28), we compute its momentum representation as

\[
\hat{\psi}(k) = \frac{1}{\sqrt{2\pi}} \left[ \exp(ir_0 \cdot k) + \exp(-ir_0 \cdot k) \right]. \tag{31}
\]

Consequently, we obtain the corresponding intensity

\[
I = \left| \hat{\psi}(k) \right|^2 = \frac{2}{\pi} \cos^2(r_0 \cdot k), \tag{32}
\]

which in essence corresponds to Eq. (5). These results link the Heisenberg uncertainty principle and classical optics; a restriction in positions \( \pm r_0 \) due to the slits results in many \( k \) values in momentum. We can extend this argument to different geometries and achieve interesting patterns in momentum space, as presented in Sec. III.

The arguments from the linear position and linear momentum representation also hold for the angular position and angular momentum domains. In this case, our apertures are “cake-like” slices that lead to an angular momentum distribution of states. In the angular case the transformations read as

\[
\hat{\psi}(\theta) = \frac{1}{\sqrt{2\pi}} \int \psi(\theta) \exp(-i l\theta) \, d\theta \tag{33}
\]

and

\[
\psi(\theta) = \frac{1}{\sqrt{2\pi}} \sum_{-\infty}^{\infty} \hat{\psi}(\theta) \exp(i l\theta), \tag{34}
\]

where \( \psi(\theta) \) represents the field in terms of its spiral harmonics, with \( \theta \in [0, 2\pi] \). The angular slits can be written as

\[
\psi(\theta) = \text{rect} \left( \frac{\theta - \theta_0}{\epsilon} \right) + \text{rect} \left( \frac{\theta - (\theta_0 + \pi)}{\epsilon} \right), \tag{35}
\]

where \( \theta_0 \) and \( \epsilon \) are, respectively, the angular location and size of the slice, while

![Fig. 14. Experimental results: (a) intensity pattern, (b) polarization map, (c) diagonal component, and (d) antidiagonal component.](image)
even. Moreover, this behaviour is modulated by a sinc$^2$ envelope, which is in agreement with the literature. The modal basis is simply the angular harmonics, or azimuthal holograms as shown in the lower portion of Fig. 15 to depict the fringe phenomenon for $l = -8$ through 8.

\[
\text{rect}(x) = \begin{cases} 
0 & \text{if } |x| > 1/2 \\
1/2 & \text{if } |x| = 1/2 \\
1 & \text{if } |x| < 1/2.
\end{cases}
\]  

(36)

By computing Eq. (33), we obtain its angular momentum representation

\[
\tilde{\psi}_l = \frac{\epsilon}{\sqrt{2\pi}} \exp(-i0_t_l) \text{sinc}(c_l/2) \left[1 + \exp(-i\pi l)\right],
\]

(37)

whose spectral distribution has the form

\[
I_l = |\tilde{\psi}_l|^2 = \frac{2\epsilon^2}{\pi} \text{sinc}^2(c_l/2) \cos^2(\pi l/2),
\]

(38)

where sinc$(x) = \sin(x)/x$. Observe that Eq. (38) is valid for discrete $l$ values and the cosine term only survives when $l$ is even. Moreover, this behaviour is modulated by a sinc$^2$ envelope, which is in agreement with the literature.

Figure 15 shows the setup to address fringes with OAM. We direct a HeNe laser beam onto the first half of a reflective SLM. In the device, two angular slits are encoded as shown in Fig. 16. After being reflected by the SLM we perform a modal decomposition\cite{32} with aid of the second half of the SLM and the subsequent Fourier transform by a thin lens. The modal basis is simply the angular harmonics, or azimuthal phases, given as the terms in the summation of Eq. (34). The purpose of the modal decomposition is to find the unknown coefficients $\psi_l$, which requires azimuthal holograms as shown in the lower portion of Fig. 15 to depict the term exp($i\phi$) in the integral of Eq. (33). To construct the OAM spectrum, we perform an inner product of these match filters with the input field. This is achieved by measuring the on-axis intensity at the focal plane for $l \in [-8, 8]$ (see Ref. 32 for additional details). Figure 17 shows the experimental results. Observe that the intensity pattern shown in Fig. 17(a) does not manifest the fringe phenomenon; the main effect observed is only diffraction due to the angular slits. Nonetheless, looking at the OAM content of the diffracted beam we notice the fringe behavior for discrete OAM values.

### VI. DISCUSSION

Interference is a venerable topic and here we have attempted to bring some of the concepts together in a holistic fashion with a modern twist. There is a misconception among some that interference is observed as fringes in intensity, which is true only for coherent light of the same polarization. Over 100 years ago Wood realized that fringes could be observed in the polarization pattern itself.\cite{20} A fun home experiment would be to try to observe the fringes seen by Wood using 3D cinema glasses. If polarization fringes exist they should be visible with the glasses on, and invisible without them (you may have to blink your eyes with the glasses on to see them). We have done precisely this in our laboratory and observed the polarization fringes reported in this manuscript. Revisiting these concepts is timely because of the topical nature of polarization research today. There is active research in developing techniques to control the spatial distribution of polarization with spatial light modulators,\cite{26} leading to exotic structured light fields\cite{27} as well as the detection of these structured fields.\cite{28,33} This structuring and detection extends to three dimensions, as demonstrated in the recent polarization M"obius strip report.\cite{30} This study also highlights the analogies between linear position and momentum and angular position and momentum. Thus, fringes can be observed in all momentum spaces: linear, spin angular momentum, and orbital angular momentum. So while the topic of interference is very old, it has recently been rejuvenated by the power of digital holograms and the topical nature of structured light fields.

It is also worth noting that interference is not limited to the classical regime but manifests in the quantum regime too.\cite{37-43} Visibility in quantum experiments are very useful measures of entanglement and quantum interference, where interference has been observed between independent photons. In these cases, the fringes are seen in coincidence counts. The framework we provide here may prove a useful starting point for students with an interest in this exciting field. The best way to learn is to do, so we include as supplementary material all the code needed to reproduce the holograms used in this study.\cite{44}

![Fig. 15. (Top) Experimental setup (M1, M2: Mirrors; SLM: Spatial Light Modulator; L: lens; CMOS: Detector). (Bottom) The azimuthal holograms used for the modal decomposition for $l = -8$ through 8.](image1)

![Fig. 16. Angular slits; (a) the desired aperture and (b) the encoded hologram.](image2)

![Fig. 17. Experimental results: (a) intensity pattern diffracted by the angular slits; (b) modal decomposition of the beam after the slits for $\epsilon = 30^\circ$.](image3)
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VII. CONCLUSION

In this report, we have outlined a general approach to analyze interference fringes in many observables, and have tried to convey the richness of the physics behind the elegance of the concepts. We have revisited this venerable topic with a modern approach by demonstrating how digital holograms may be used as an enabling tool in the laboratory for interference experiments. We have provided a tutorial approach, complete with code to reproduce the holograms, to allow easy implementation of the core ideas. As structuring light fields is a highly topical research field, this work may be a useful foundation for students wanting to learn the basics first hand.


1. Gossman et al.

4 T. Young, A Course of Lectures on Natural Philosophy and the Mechanical Arts, A Course of Lectures on Natural Philosophy and the Mechanical Arts No. v. 1 (Johnson, London, 1807).

See supplemental material at http://dx.doi.org/10.1119/1.4948604 for Matlab codes.