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Propagation of orbital angular momentum carrying beams through a perturbing medium

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Abstract

The orbital angular momentum of light has been suggested as a means of information transfer over free-space, yet the detected optical vortex is known to be sensitive to perturbation. Such effects have been studied theoretically, in particular through turbulence. Here we demonstrate a simple apparatus to introduce turbulence-like distortions to optical fields propagating over a long path. We create vortex beams and observe their propagation through a heated spinning pipe, known to mimic the two primary atmospheric aberrations, namely tip–tilt and defocus. We use a digital decomposition tool to modally resolve the distorted vortex beam into its azimuthal components to observe the impact of the medium on the detection of the encoded vortex charge. Such techniques are useful in studies of free-space optical communication with orbital angular momentum.

Keywords: orbital angular momentum, turbulence, GRIN, vortex

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(Some figures may appear in colour only in the online journal)

1. Introduction

Photons have been known for over a century to carry spin angular momentum through circular polarization [1, 2], however orbital angular momentum (OAM) and its relationship to light has been a much more recent field of investigation [3, 4]. Various fields of optics have exploited the discovery that light beams with an exp(\(i\ell\theta\)) azimuthal phase component carry integer OAM of \(\ell\hbar\) per photon [5], ranging from optical trapping and tweezing [6–8], to quantum communication [9–11] and astronomy [12]. In particular, the field of free-space optical communication (FSOC) has recently begun to exploit the large dimensionality of the OAM degree of freedom as a means of increasing data storage density in light [13, 14].

One of the problems with FSOC encoding information in azimuthal modes is that the optical path always includes some degree of optical turbulence [15], which has a deleterious effect on the mode purity. It has been shown theoretically that OAM-carrying beams are particularly sensitive to turbulence [16–19]. This can be understood intuitively because (i) all spatial modes are affected by the aberrations introduced by the atmosphere, and OAM-carrying beams are no exception, and (ii) OAM is an extrinsic property of the field (as compared to the intrinsic property spin angular momentum) and hence the amount of OAM measured is determined by the axis around which the measurement is made [20]. Some experiments on OAM modes through turbulence have been conducted at both the classical and at the single photon level, where the atmosphere has been approximated as a ‘thin’ phase screen using a spatial light modulator as a turbulence simulator [21, 22].

In this study we make use of a low-budget and an easy to implement device in the form of a heated, spinning, metal
pipe to create a ‘thick’ perturbing medium in the laboratory. The pipe introduces extensive tip–tilt and defocus distortions [23, 24] and under stable conditions acts as a gradient-index (GRIN) medium, but instabilities in the flow result in random density fluctuations that have been suggested to follow the van Karman atmospheric turbulence model [25]. The device is tunable insofar as perturbations can be controlled by specifying the pipe wall temperature and pipe rotation speed. We pass ideal vortex beams through the medium and experimentally observe the distortions to the measured azimuthal mode content. In particular we observe beam wander, defocus and intensity distortions after propagating through the medium, and we show that these aberrations have a deleterious impact on the detection of the OAM modes. This models, on a small scale, the effects of atmospheric turbulence on OAM modes that must be corrected in order to make OAM-encoded free-space optical communication practical.

2. Vortex beams

2.1. Creating vortex beams

OAM-carrying beams, sometimes referred to as ‘vortex beams’, have a helical phase structure and can be characterized by the number of radii along which the phase is 0 in one cross-section. Geometrically \( \ell \) corresponds to the helical winding index, or topological charge of the vortex beam. The example shown in figure 1 has a winding index of \( \ell = 3 \), corresponding to OAM per photon of \( 3 \hbar \). Such a beam would have an optical vortex charge \( \ell = 3 \).

The OAM of light is closely related to the spatial structure of the field, which must satisfy the paraxial wave equation in a given geometry. The intensity vanishes at the centre of the field because of the phase singularity at this point (see figure 1). Thus all OAM-carrying modes have null intensity on the axis aligned with the direction of propagation, causing the intensity profile to appear similar to a doughnut. The general form of the mode is given by

\[
\psi^{\ell}(r, \theta) = A(r) \exp(i\ell\theta) \tag{1}
\]

where \( A(r) \) is a general radial function and \( \theta \) is the azimuthal coordinate. The form of the radial function determines the type of mode:

\[
A_{LG}(r) = a_p r^{l_2} J_{l_1} \left( \frac{2r^2}{w^2} \right) \exp \left( \frac{-r^2}{w^2} \right) \tag{2}
\]

\[
A_{BB}(r) = a_l J_l(k_l r) \tag{3}
\]

Equation (2) is the radial component for a Laguerre–Gaussian mode [26], where \( a_p \) is a normalization constant, \( l_2 \) is an associated Laguerre polynomial and \( w \) is a scale parameter related to the Gaussian beam width. Equation (3) is the radial component for a Bessel beam, where \( J_l \) is a Bessel function of the first kind and \( k_l \) is the radial wavevector. In this work we will restrict ourselves to the Laguerre–Gaussian \( LG_{p}^{\ell} \) modes.

\( LG_{p}^{\ell} \) beams can be formed using a variety of different methods that differ in terms of the beam purity and intensity. We choose to generate \( LG_{p}^{\ell} \) beams using a phase-only spatial light modulator (SLM) because of the demonstrated high purity of these beams and the ease with which beam characteristics can be adjusted digitally [27]. As the function in equation (2) requires both amplitude and phase specification, complex amplitude modulation [28, 29] is employed to create the desired modes from an incident Gaussian field. An illustration of the transformation of a Gaussian mode into a vortex mode is shown in figure 2, along with the required hologram displayed on the SLM.

This method for generating beam profiles can be extended to create superpositions of vortex modes. For example, a superposition of \( LG_{0}^{5} \) and \( LG_{5}^{0} \) creates a petal-like field with a net OAM of 0. Figure 3 demonstrates experimentally measured modes of an \( l = 5 \) mode, an \( l = 5 \) and \( -5 \) mode superposition, an \( l = 9 \) mode and an \( l = 9 \) and \( -9 \) mode superposition created using this approach. The value of \( C \) indicates the normalized correlation of the experimental and theoretical beam intensity profiles. The purity of the created
Figure 3. Experimentally generated modes with theoretical intensity (upper inset) and hologram (lower inset). (a) LG50 (C = 94%), (b) superposed LG50 and LG−50 (C = 90%), (c) LG90 (C = 94%), (d) superposed LG90 and LG−90 (C = 90%).

modes was high (typically between 90 and 95%) but not perfect, due to the extended path length over which the experiments were performed, thus assigning an approximately 5% mode error to the creation process (see section 2.2).

2.2. Detecting vortex beams

To decode the information stored in OAM modal content, the receiver must have a method of decomposing the field into azimuthal modes. A number of methods, including interfering a plane wave with the received wave [30], examining the diffraction pattern after predetermined apertures [31], and more recently using a geometrical optical transformation [32], have been developed accordingly. We choose to find this decomposition using an optical inner product [33], by means of an SLM and a lens, with the azimuthal harmonics as an orthonormal basis [34–36]. In this vein, the optical field, $U(x)$, is decomposed into a set of basis functions, or modes $\psi_n(x)$, each weighted with a complex expansion coefficient. The task is to find these coefficients that incorporate the modal weightings ($\rho_\ell^2$) and their phases ($\Delta \phi_\ell$) such that

$$U(r, \theta) = \sum_{\ell=1}^{\ell_{\text{max}}} c_\ell \psi_\ell(r, \theta),$$

where the unknown coefficients of the basis functions

$$c_\ell = \rho_\ell \exp(i\Delta \phi_\ell) = \langle \psi_\ell | U \rangle$$

can be uniquely determined by the orthonormal property

$$\langle \psi_n | \psi_m \rangle = \int_{\mathbb{R}^2} d^2 x \, \psi_n^* (x) \psi_m (x) = \delta_{nm}. \quad (6)$$

A full description of the spatial mode requires at least two indices (e.g., the $p$ and $\ell$ indices of the LG basis). Since we restrict ourselves to an azimuthal basis, resulting in the well-known spiral spectrum with its width defined by the spiral bandwidth, the complete expansion can be written as

$$U(r, \theta) = \sum_{\ell=1}^{\ell_{\text{max}}} c_\ell (r) \exp(i\ell \theta). \quad (7)$$

Here $| c_\ell (r) |^2$ is the relative power in the corresponding $\ell$ mode. In order to find the amplitude of a specific mode $\ell$ the conjugate mode $\exp(-i\ell \theta)$ is encoded as the transmission function of the SLM so that the intensity in the focal plane on the optical axis is a measure of the power in that mode. An experimental setup for this decomposition is shown in figure 4, with the sample inner product $\langle \psi_6 | \psi_7 \rangle$ optically measured.

By computing the inner product with a range of $\ell$ modes a full decomposition can be obtained. Experimentally a circular 0.05 mm radius aperture was centred on the optical axis and used to compute the relative power in each mode. By using a multi-pixel region as the basis for the decomposition,
2.3. Influence of beam perturbations

It is clear that in general a complete, high-quality decomposition requires that both the scale of the basis and its optical axis are known [37]. Scaling-related measurement distortions are demonstrated in the following example, where an optical field $U$ is decomposed into a Laguerre–Gaussian set of two sizes: the correct scale, $w$, and an offset scale, $\tilde{w}$:

$$U(r, \theta) = \sum_{\ell=1}^{\ell_{\text{max}}} c_{\ell}(r; w) \exp[i\ell\theta].$$

By inspection of equations (2) and (8) it is evident that the modal spectrum changes with the scale of the basis set; since $c_{\ell} \neq \tilde{c}_{\ell}$ the measured LG basis spectrum differs from the encoded one. The scale-dependence of the decomposition is addressed by the choice of azimuthal harmonics as the orthonormal basis, wherein the scale information is stored in the coefficients and the basis functions depend only on $\ell$ and $\theta$ (i.e. no dependence on scale $w$). However an offset in beam axis position corresponds to both a radial and an azimuthal offset, so that

$$U(r - r_0, \theta - \theta_0) = \sum_{\ell=1}^{\ell_{\text{max}}} c_{\ell}(r - r_0) \exp[i\ell(\theta - \theta_0)]$$

and thus in this case the measured OAM spectrum itself is perturbed.

An extensive theory has been developed to explore what will be measured if a beam is decomposed on an axis other than its axis of symmetry, and how this differs from the on-axis decomposition [20]. Figure 7 models the impact of beam axis wander and size defocus during the decomposition process, for the general case of a decomposition into the full LG spectrum. Wander and defocus are the most significant optical aberrations introduced by atmospheric turbulence.

3. Simulating turbulence-like perturbations

An empirical understanding of how atmospheric turbulence gives rise to OAM modal coupling is fundamental to...
Figure 6. Multiplexed holograms allow multiple channels to be detected simultaneously (sample result for the inner product with $\ell = 4–8$ modes). (a) Channel power represented by intensity at grid vertices, (b) incident $\ell = 7$ ($C = 92.4\%$) mode.

Figure 7. Effects of defocus and wander on the LG$_{p}^{l}$ basis decomposition of an incident LG$_{50}^{50}$ mode. (a) Spreading of the azimuthal modes due to $\frac{1}{2}w$ beam axis offset (wander), (b) spreading of the radial modes due to $\frac{1}{2}w$ defocus, (c) simultaneous wander and defocus of $\frac{1}{2}w$ results in severe broadening of both the radial and azimuthal mode spectra. The inset in (c) shows the intensity of the incident beam.

the implementation of OAM-encoded FSOC. This coupling widens the received OAM spectrum as in figure 7. Numerous models, both mathematical and experimental, of atmospheric turbulence have been developed [38, 39]. A common implementation is that of a single phase screen written to an SLM to mimic the entire optical path through turbulent
conditions [21]. This is for the most part a reasonable approximation for weak turbulence conditions (e.g. over short distances or stable atmospheric regions) but has shortcomings when the turbulence becomes strong [15, 18]. Recently [22] a two-screen approach has been suggested and used with success to simulate a 1 km path through turbulence.

Here we mimic the turbulence-like perturbations using a spinning heated pipe (SPGL), externally heated and rotated at high-speed. The benefit of this laboratory model is that, similar to atmospheric turbulence, the SPGL exhibits optical aberrations composed largely of the lower-order Zernike polynomials, namely, tip–tilt and defocus. Moreover, the SPGL allows for a long path, is low-budget, and is easily controlled and implemented. Unlike SLMs, there are no resolution restrictions, and the aperture size can be made very large. This makes it an ideal device for implementing strong turbulence regimes.

3.1. SPGL characterization

The SPGL is a GRIN medium wherein the exchange of heated air inside the pipe and cooler air outside the pipe results in a density gradient both across the diameter of the pipe and along its length. We propose that such a device can be used to generate turbulence-like conditions in the laboratory, the strength of which depends on both the SPGL’s rotation speed and temperature. The mixing of hot and cold gases generates turbulence and concomitant phase distortions on a passing optical field. Previously we have studied the phase distortions in an effort to improve the SPGL as a high-quality lens, both from an optical and computational fluid dynamics (CFD) approach [23, 24]. Here we report how the pipe action may be characterized by the modified von Karman standard atmospheric turbulence model, and illustrate that the turbulence strength may indeed be controlled over several orders of magnitude with reasonable adjustment of the pipe rotation speed and wall temperature. The SPGL is therefore a useful tool for introducing controlled and repeatable (statistically speaking) aberrations to propagating light.

Our approach to characterizing the aberrations and turbulence in the SPGL is based on the angle-of-arrival slope correlation method, details of which can be found in [25, 40, 41]; a brief description of the experiment and some typical data is given in the appendix. Our results are summarized in figure 8. The graph shows that, in general, the strength of the turbulence increases with rotation speed for a constant temperature according to a general proportional rule, \( C_n^2 \Delta L \propto \omega^3 \), where \( C_n^2 \) is the turbulence refractive index structure constant, \( \Delta L \) is the path length (pipe length in our experiment), and \( \omega \) is the pipe rotation speed in Hz. It is clear that the turbulence strength ranges over three orders of magnitude. Importantly, the SPGL mimics strong turbulence conditions.

4. Vortex beam perturbation results

Our SPGL comprises a 1.43 m stainless steel pipe with an internal diameter of 3.66 cm. The pipe was electrically heated with commercial heating tape to a maximum temperature of 150 °C and rotated with the aid of a pair of simple sewing machine motors; the rotation speed of the pipe was measured with an optical sensor. The chosen motors do not allow bi-directional rotations so experiments were conducted uniquely with the pipe rotating clockwise. The SPGL was rotated in two regimes: a constant rate of 15 Hz (regime 1) and a rate varying sinusoidally from 10 to 15 Hz with a frequency of 0.5 Hz (regime 2). A HeNe laser beam was expanded and modified by suitable digital holograms to create the desired vortex beams with flat wavefronts at the entrance plane of the pipe. The size of the second moment radius was set to be the same for all orders of the vortex beams chosen, \( w = 2.2 \, \text{mm} \), so that the Rayleigh range exceeded the pipe length for all \( \ell \). These ‘collimated’ vortex beams were carefully aligned through the pipe, and then relay imaged to a second SLM for the modal decomposition. The beams could also be directed to a CCD camera to observe the change in intensity pattern.

4.1. Beam wander

The main contribution to beam perturbation through the atmosphere is tip and tilt on the wavefront [42]. This is also the primary aberration caused by the pipe when it is heated but not rotating. Convection currents will act such that the higher density air moves to lower areas of the pipe, creating a temperature gradient which causes the beam to bend towards the higher density region. To test this, a Gaussian beam (also with beam waist \( w = 2.2 \, \text{mm} \)) was passed through the pipe. When the pipe was unheated the beam’s centroid on the CCD was normalized to \((x,y) = (0,0)\), and after heating to 150 °C and reaching steady state the beam’s centroid had shifted to \((-20,-320) \, \mu \text{m}\). This demonstrates that, as predicted, the heating causes the beam to tip significantly in the \(-y\) direction. When the SPGL is rotated the density gradient (on average) becomes more symmetric about the centre, but random flow instabilities contribute significantly to tip and tilt errors on the wavefront [25].
Figure 9. Beam centroid motion two-dimensional plot (left) and histogram (right) with 10 μm binning. Relative to the pipe x is the horizontal axis (blue) and y the vertical axis (red). (a) Regime 1, \((\mu_x, \mu_y) = (71.4, -29.3)\) and \((\sigma_x, \sigma_y) = (10.2, 28.5)\). (b) Regime 2, \((\mu_x, \mu_y) = (72.6, 78.5)\) and \((\sigma_x, \sigma_y) = (9.4, 44.7)\). Mean position \(\mu\) measured relative to the centroid when the pipe is stationary and standard deviation \(\sigma\), both given in μm.

the position of the beam centroid for regimes 1 and 2, where both graphs are normalized to the mean position of the beam in that regime. Note that the tip–tilt error was tested with the Gaussian mode, although it applies equally to all the vortex fields as this centroid movement is not mode dependent.

In both regimes the beam wander in the y direction is very pronounced relative to that in the x direction. In addition the mean position of the beam centroid is significantly displaced from its normalized position when the pipe is heated but not rotating. We notice that in all cases there is a significant beam wander effect, with a large standard deviation in both the x- and y-coordinates of the beam centroid.

The standard deviation in the x-centroid is very similar for both regimes, while the standard deviation in the y-centroid for regime 2 is on the order of 1.5 times greater than that for regime 1. This supports the expectation that varying the rotation rate results in additional instabilities in the density gradient in the pipe and so strongly influences the y-centroid position, whereas the x-centroid position variation is only weakly dependent on the rotation rate. The large beam wander in the mean position from the propagation axis prior to pipe rotation implies a mismatch between the encoded OAM spectrum and the measured OAM spectrum, as shown in section 2.3. Even if the average offset is corrected for, the random motion will distort the measured OAM spectrum unless a high-speed, active detector is implemented to track the beam axis.

4.2. Beam defocus and distortions

Along with the motion of the centroid of the beam the SPGL also causes significant defocus effects. Defocus is the second most significant contribution to wavefront error due to atmospheric turbulence [42]. This is also the primary aberration caused by the pipe when it is heated and rotating at a steady rate. Convection currents will act such that the higher density air is near the centre of the pipe while the lower density air is at the pipe walls, such that on average the pipe acts like a GRIN lens [23]. Due to the flow instabilities, there are random higher-order aberrations introduced which distort the beam.

To demonstrate clearly distinguishable beam defocus the rotation rate of the pipe was increased continuously from 0 to 15 Hz, as shown in figure 10 for modes \(l = 5, 7\) and 9 (see video 1 available at stacks.iop.org/JOpt/15/105706/mmedia for \(l = 7\) mode defocus) at a temperature of 150°C. After correcting for centroid variation, the beam is focused due to the GRIN lens nature of the SPGL. The transition to instability occurs when the rotation rate rapidly increases or decreases as in regime 2 rotation, or at the extreme of very fast rotation. In regime 2 the effects are particularly distinguishable because there are alternating phases of increasing and decreasing rotation rate. As the rotation rate increases, the focusing properties of the SPGL are more apparent but the beam profile remains relatively undistorted until higher rotation rates introduce the onset of random instabilities. At transitional rotation rates, turbulence within the pipe generates a variety of
higher-order aberrations. We can quantify this by considering the correlation between the time series of measured intensity profiles and the desired profile. To consider the effect of the higher-order aberrations, we compensate in the correlation calculation for the effects of beam wander and beam defocus (beam width). The results for the changing correlation as a function of time are shown in figure 11 for an $\ell = 7$ mode passing through the pipe under conditions of regime 2 rotation (see video 2 (available at stacks.iop.org/JOpt/15/105706/mmedia)).

Given the spectrum of aberrations introduced in a beam propagating through turbulent conditions, the modal decomposition is severely distorted, as shown in figure 12 for regime 1 flow. Experimentally 16% of the received power is measured to be contained in the transmitted mode, and a far larger fraction (47%) is contained in mode $l = 9$. Due to the random motion of the beam at the decomposition plane the decomposition axis cannot be maintained in alignment with the incident beam axis, resulting in fluctuations in the measured decomposition independent of the actual modal content of the beam. Figure 12 clearly demonstrates that although the decomposition axis is normalized to the position of the beam centroid when the pipe is heated, this method cannot effectively be used to recover the information contained in the $\text{LG}_7^0$ transmitted beam.

Experiments with flat plate approximations of turbulence have noted that in cases of strong turbulence the peak received power can be shifted to a mode other than the transmitted mode. Anguita et al observe this occurrence for an $l = 1$ transmitted mode with a turbulence refractive index structure parameter of $C_n^2 = 10^{-13}$, although it is suggested that this effect is likely restricted to lower $l$ modes [39]. The results
in section 2.3 indicate that for rotation rates of 10–15 Hz and a wall of temperature 150 °C, \( c_n^2 = 10^{-8} \) by order of magnitude (see section 3.1). Hence the regime under study is significantly more turbulent than that achieved with thin phase approximations and demonstrates shifting of peak power to non-transmitted modes even in cases of large transmitted \( l \) values. The SPGL is thus well suited to simulating strong turbulence conditions, e.g., long paths or propagating fields through horizontal paths near a surface. Most single screen approximations do not realize such conditions.

5. Conclusion

We have experimentally investigated the behaviour of an OAM-encoded beam passing through a heated, rotating pipe. This device is low-budget, easy to implement, and mimics the primary aberrations found in the atmosphere. Moreover, with a long path length, which may be made arbitrarily longer by multiple passes of the light through the device, it affords the experimenter a simple means to mimic thick medium propagation in addition to the thin medium effects simulated with spatial light modulators. We used the SPGL to demonstrate the significant tip–tilt and defocus on the OAM modes detected, as well as mode distortions due to higher-order aberrations, and modelled their effect on modal decomposition. The random fluctuations of beam position and size were shown to cause severe deterioration of the beam quality and to render measurements of the modal spectrum unreliable. The results of these experiments lend credence to the need for alternative OAM measurement techniques, either using active components to track the beam axis in real time or transmitting a reference beam along with the signal to allow for receiver normalization. Resolving the challenge of effectively decoding OAM channels through strong turbulence will make possible long-range experiments with OAM-encoded signals, and the establishment of the SPGL as an appropriate laboratory model of strong turbulence may facilitate preliminary advances in this field.

Appendix

An expanded beam from a HeNe laser was transmitted along the axis of the SPGL and incident on a Shack–Hartmann wavefront sensor used to measure the local tip and tilt. The sensor had a detector grid of 69 × 69 microlenses of size 7.4 mm × 7.4 mm. Data was acquired at a rate of approximately 1 Hz, insufficient for an accurate temporal analysis but allowing determination of the primary aberrations along with slope maps for the horizontal and vertical directions respectively. A range of pipe wall temperatures and rotation speeds were tested. The slope maps are directly related to the power spectral density of the turbulence, which is in turn determined by the turbulence model used and the strength of the turbulence. By applying a fitting routine to the slope maps, the turbulence information of the SPGL could be inferred. An example of data from a typical measurement is shown in figure A.1. The slope data was measured for various points \((x, y)\) across the pipe area (averaging over its length), and the correlation between points at various displacements (distances apart) calculated. This was done for both the horizontal (\(x\)) and vertical (\(y\)) directions, and for rows (\(r\)) and columns (\(c\)) of data in the \(xy\) grid. The data was best described by the von Karman spectrum [15, 25]; a nonlinear least squares fit to the experimental data with this model provided the turbulence parameters reported in the main text. The agreement between the fitted model (th) and experimental data (ex) is shown in figure A.1 and is clearly very good.

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