Angular self-reconstruction of petal-like beams

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The self-reconstruction of superpositions of Laguerre–Gaussian (LG) beams has been observed experimentally, but the results appear anomalous and without a means to predict under what conditions this take place. In this Letter, we offer a simple equation for predicting the self-reconstruction distance of superpositions of LG beams, which we confirm by numerical propagation as well as by experiment. We explain that the self-reconstruction process is not guaranteed and predict its dependence on the obstacle location and obstacle size.

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That some optical fields may self-heal, or self-reconstruct, is now well known, having first been discovered and studied in the context of Bessel modes [1,2] and their superpositions [3]. In such cases, the self-healing is understood as the interference of plane waves, traveling on a cone, that bypass the obstacle. The reconstruction distance in this instance is determined from geometric arguments.

More recently, it has been recognized that there are other classes of optical fields that self-reconstruct [4], and interestingly also the Laguerre–Gaussian (LG) modes [5]. It was shown experimentally that such LG modes do, at least in some instances, reconstruct after an obstacle, but at present there is no means to predict under what conditions or to what extent the reconstruction will take place. In this Letter we offer a simple concept for the reconstruction of superpositions of LG beams based on the handedness of the modes and the rotation of the Poynting vector. While these concepts are not new, we apply them for the first time, to the best of our knowledge, to derive, from geometrical principles, an expression for the angular self-reconstruction distance after an obstacle. We show that reconstruction is not guaranteed and is influenced by the distance between the obstacle and the waist plane of the LG modes interacting with it. We confirm the model both numerically and experimentally.

By way of example, consider the propagation properties of a superposition of two azimuthal LG modes, so-called petal modes [6,7] or optical Ferris wheels [8], of opposite helicity and direction of the Pointing vector. The electric field for such a superposition may be written in the general form

$$u(r,\phi) = A(r)[\exp(il\phi) + \exp(-il\phi)], \tag{1}$$

where (r,ϕ) are the coordinates, $A(r)$ is a general radial enveloping function, and l is the azimuthal index. Each mode rotates during propagation by an amount given by [9,10] $\theta = \arctan(z/z_R)$, independent of the azimuthal index l, and where the propagation distance (z) from the waist plane is normalized to the Rayleigh range, z_R. This rotation effect has been observed experimentally [9]. In Fig. 1(a) we show a numerical simulation of the propagation of two obstructed LG beams with opposite helicity and thus differing direction of the orbital angular momentum vector. We notice that the obstructed areas of these beams effectively rotate in opposite directions. Intuitively it appears that regions of obstructed light from one mode eventually overlap with regions of unobstructed light from the other mode. Since this is true for each component of the superposition, such an obstructed area in the initial plane will be self-reconstructed on propagation. It is clear that this "angular self-reconstruction" distance will depend on the angular size of the obstruction.

To derive a simple expression for this self-reconstruction distance, we recall that the maximum rotation angle of the Poynting vector for any vortex beam is $\pi/2$ [9], thus limiting the maximum angular size of the obstruction for angular self-healing. This maximum rotation is based on propagation from the waist plane to infinity. In the case of an obstruction that is not at the waist plane, the maximum rotation angle will decrease to $\theta = \arctan((z+z_I)/z_R) - \arctan(z_I/z_R)$, where the distance to the waist plane is z_I. Let us assume that total reconstruction is achieved when the angular rotation of each component of the field exceeds the angular extent of the obstruction: $\theta > \theta_I$. An example of an obstructed beam is shown in Fig. 1(b), with an angular extent of θ_l. We have placed the obstacle at a distance of $r_p = w_g\sqrt{l/2}$ from the beam center since the peak intensity of azimuthal LG beams (and their superpositions) is found on a ring of this radius (where w_g is the Gaussian beam size), but it is only the angular extent that matters. Following this argument, the angular self-reconstruction distance, $z_{\min}$, can easily be found to be

$$z_{\min} = z_R \operatorname{Tan}\left(\theta_I + \arctan\left(\frac{z_I}{z_R}\right)\right) - z_I. \tag{2}$$

We see that the reconstruction distance depends linearly on the Rayleigh range. The initial position of the obstacle influences the reconstruction process significantly; namely, if the obstacle is placed a at distance

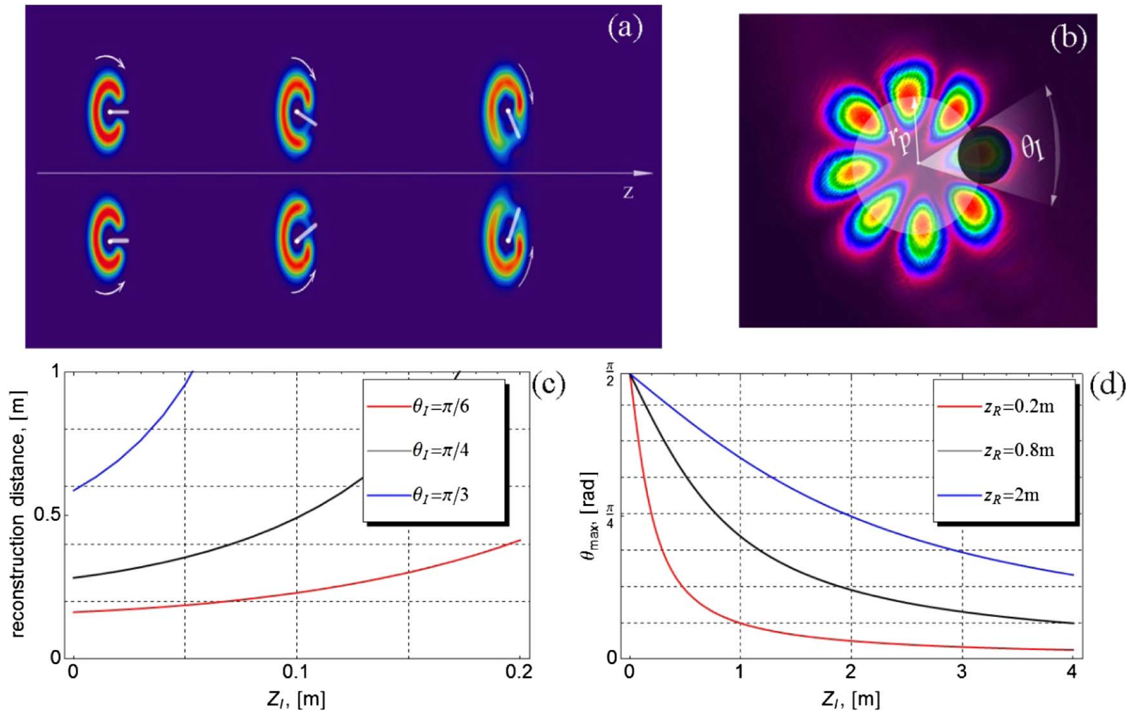

Fig. 1. (a) Schematic representation of the rotation of the shadow region in an obstructed LG beam with different signs of the angular momentum. (b) Schematic for the derivation of the self-reconstruction distance z_r and (c) dependence of the reconstruction distance on the initial position (z_I) and angular size (θ_I) of the obstacle. (d) Dependence of the maximum angular size obstruction $\theta(z_I)_{\max}$ on the initial position of the obstacle z_I for the different Rayleigh range of the beam.

equal to the Rayleigh range, then angular self-healing will fully reconstruct the initial field only if the angular obstruction is less than $\pi/4$. In Fig. 1(c) we have represented the dependence of the self-reconstruction distance on the initial position (z_I) for the different angular sizes of the obstruction.

From Eq. (2) we can find the maximum angular size for the obstruction that can be reconstructed:

$$\theta(z_I)_{\max} = \frac{1}{2}\left(\pi - 2\,\arctan\left(\frac{z_I}{z_R}\right)\right). \tag{3}$$

We see that the maximum angular size of the obstruction decreases with the distance to the waist plane and drops twice (to $\pi/4$) at the Rayleigh range distance [see Fig. 1(d)].

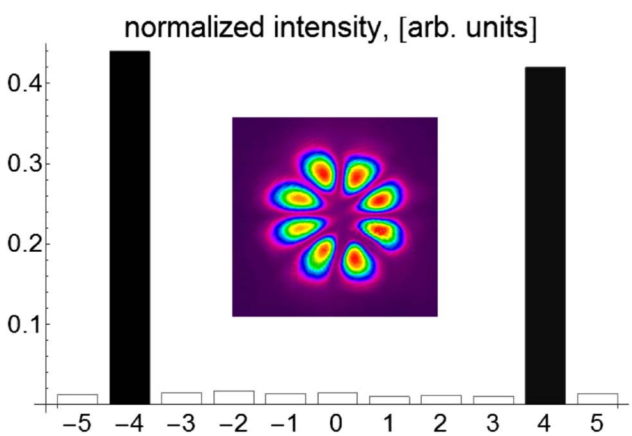

Fig. 2. Modal decomposition of eight-petal beam.

Experimental verification was carried out using an intracavity generation technique for such petal beams [6,7] but implemented with a digital laser setup [11].

The laser output was a superposition mode, shown in Fig. 2, of equal weightings of two azimuthal modes. The

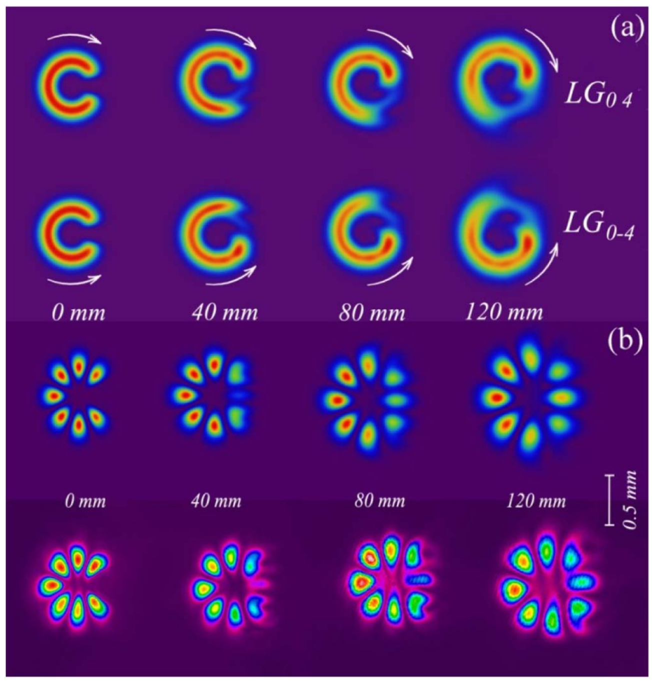

Fig. 3. (a) Simulation of the free space propagation of obstructed LG_{04} and LG_{0-4} beams. (b) Simulation and corresponding experimental verification of the reconstruction of the superposition beam (LG_{04} and LG_{0-4}).

infrared laser beam (1064 nm wavelength) was relay imaged to a waist plane with beam waist radius $w \approx 300$ μm. A $d = 200$ μm diameter obstacle was set at this plane ($z_I = 0$) to overlap with one of the petals, as illustrated in Fig. 1(b), for an angular obstruction angle of $\theta_l = 27$ deg. Using Eq. (2) we predicted a self-reconstruction distance of $z_{\min} \approx 140$ mm. We numerically propagate the obstructed field and show the impact of the obstruction on each LG mode individually as well as the superposition, shown in Figs. 3(a) and 3(b), respectively, as a function of distance.

In Fig. 3(b) we present a simulation and an experimental verification of the self-reconstruction, with the two in excellent agreement. The results also confirm the analytical expression of Eq. (2). We see in Fig. 3(b) that the petal that was obstructed in the waist plane of the beam has reconstructed totally as predicted.

In this Letter, we have presented an intuitive argument for the self-reconstruction of petal-like beams, and derived a simple analytical equation for the self-reconstruction distance. Our analysis explains previous anomalous observations [5] of why some superpositions appeared to self-heal, while others did not: we note that the self-reconstruction distance is independent of the azimuthal orders in the superposition, but depends on the distance to the waist plane of the petal-like beam. Indeed, there are conditions under which it is not possible to self-heal, for example, the maximum obstruction size drops by a factor of 2 to $\pi/4$ when placed at a Rayleigh length from the waist, decreasing further thereafter. Our new results, summarized in Eqs. (2) and (3), allow these properties to be calculated for any superposition.

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